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AN

ELEMENTARY PHYSICS

FOR SECONDARY SCHOOLS

BY

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PART I: PRINCIPLES
PART II: LABORATORY EXERCISES

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PREFACE.

THAT there is a demand for a new School Physics recent correspondence with over fifteen hundred secondary schools abundantly shows. That there are good books now before the educational public cannot be denied. The good books — those reasonably accurate, scientific, and modern — are too difficult, too diffuse, or require more than the usual school equipment of apparatus. In the so-called attempt to "enrich the grammar school courses," physics has been in a few cases introduced, and a moderate demand created for very elementary books. Thus the instructor has been left to a choice between books too elementary and simple, and those that make too great a demand upon the pupils in the usual secondary school course.

The author's experience as a teacher of physics, both in secondary schools and college, has given him exceptional opportunity to know the capacity of the average student. His relations with a large number of teachers of elementary science have been such as to make him aware of the limitations under which teachers work, both as to specific preparation for the work of teaching physics and still more as to material equipment.

The book has been written, therefore, with the needs and limitations of the average teacher and pupil constantly in mind. Illustrations are drawn, wherever possible, from facts already known to the pupil.

The illustrative experiments described in the text are few in number and require only simple apparatus. For teachers who have more elaborate apparatus at command the *accompanying handbook* gives full references to text-books which are in every teacher's library.

The plan of relegating to the handbook all matter useful mainly to the teacher relieves the book of much fine-print lumber, and allows the text to proceed without interruption in a connected and logical treatment of the subject.

The same end is still further subserved by putting certain illustrative experiments in the lists of exercises for review which follow each chapter, and by putting the laboratory exercises by themselves in Part II.

Flexibility in amount of work done is attained by giving a large number of exercises, some of which are rather difficult. Of the *seventy-five laboratory exercises in Part II, but forty would be required for entrance to Harvard.* Of the exercises in Part I, most classes would probably perform about half. The text should all be mastered by the pupil as far as he goes.

The work in Part II — which contains many new and practical ideas — *should begin the second week of the term and continue parallel with work on the text.* The experiments in Part II are all quantitative and lead to

results which may be easily verified from data in the teacher's hands. The tendency of the best teaching practice is growing rapidly in the direction of leaving qualitative experiments to the class room to be performed by the teacher, or under his direction, and giving to the pupil experiments involving the law which he has already learned from the book. To calculate the breaking stress of one or two particular wires from measurements of the diameters of the wires and the force required to break them, and to find that the value obtained compares fairly well with the results obtained by others as given in the book, is good proof of the correctness of the law. Careful weighing with the steelyards and balance illustrates the law of moments perfectly, while giving the student a much better training in accuracy than he will get by using levers supported below their centres of gravity, as suggested in many books. Most of the experiments in Part II have been tested during the past three years in a number of schools, including both the large city high school and the small country school, with most gratifying results.

The order of arrangement of the chapters in Part I demands a word of explanation. The author's observation has been that the student encounters so many difficulties in the early part of the study of mechanics that he is likely to become discouraged before his interest is fully aroused in the subject. He is hardly familiar with the idea of force before the more complex

idea of energy is thrust at him and he becomes confused. Moreover, he is expected to comprehend the law of conservation of energy when he has studied but one form of energy.

By first treating all forms of energy as forms of motion, or capable of being converted into motion, the subject of energy is deferred until the student is in possession of a large body of facts which bear directly upon it. Moreover, the importance of a thorough knowledge of mechanics is emphasized by bringing it directly into relation with heat and electricity.

It is believed that the treatment of wave motion here given is more connected and clear than is usually found in elementary text-books. The subject of index of refraction is treated with scientific accuracy without the introduction, either implicitly or by name, of the trigonometric functions (pages 204 and 351). It is the author's firm conviction that treatments involving any knowledge of mathematical notions beyond those gained in elementary algebra and plane geometry are out of place in a text-book designed for secondary schools. The definition of index of refraction as a ratio of the velocities of light in the two media calls attention to the physical fact involved, while its definition as the ratio of the sines of two angles directs attention to one of the methods usually employed in its determination.

Teachers are requested in examining the book to read the corresponding portions of the handbook in connection with the text.

Corrections or suggestions from teachers using the book will be gratefully received by the author, who realizes that the task of writing a book which shall be at once simple and accurate is by no means an easy one.

I am under obligations to Mr. Frank Wenner, Instructor in Knox Academy, to Professor F. L. Bishop, of Bradley Polytechnic Institute, to Mr. Allan C. Rearick, Instructor in Science in Kewanee High School, and to Mr. L. E. Flanegin, Superintendent of Schools, Elmwood, Ill., who have rendered valuable assistance in the preparation of the book, either in the way of suggestions or in reading the proof, and to Mr. Wenner and Mr. G. A. McMaster for work upon the drawings.

C. B. T.

GALESBURG, ILL., January, 1900.

CONTENTS.*

PART I. — PRINCIPLES.

INTRODUCTION.

CHAPTER I.

MATTER AND MOTION.

CHAPTER II.

BALANCING FORCES.

SOME PROPERTIES OF MATTER.

* The references are to pages.

CHAPTER III.

HEAT.

CHAPTER IV.

ELECTRICITY AND MAGNETISM.

CHAPTER V.

WORK. ENERGY. MACHINES.

CHAPTER VI.

VIBRATIONS. WAVES.

CHAPTER VII.

SOUND.

CHAPTER VIII.

LIGHT.

PART II.—LABORATORY EXERCISES.

INTRODUCTION.

CHAPTER IX.

LENGTH.

CHAPTER X.

MASS.

CHAPTER XI.

TIME.

CHAPTER XII.

FORCE.

CHAPTER XIII.

HEAT.

CHAPTER XIV.

ELECTRICITY.

CHAPTER XV.

SOUND.

CHAPTER XVI.

LIGHT.

PHYSICS.

PART I. — PRINCIPLES.

INTRODUCTION.

1. The Place of Physics among the Sciences. — Natural Science has for its aim the explanation of natural phenomena — the things which happen about us every day. It was originally all comprised under Natural Philosophy, or Physics. Our knowledge of natural phenomena has so increased during the last few centuries that no one man could hope to master all of it. Natural Science has been divided, therefore, into several branches. Astronomy deals with phenomena wholly beyond our control. Chemistry studies changes which result in the formation of new substances. Biology deals with the phenomena manifested in living things. *Physics* is now confined to the study of those changes which occur in the position, appearance, or state (solid, liquid, gaseous) of bodies; it seeks to trace each change to an adequate cause and to discover the laws which govern the phenomena observed. Physics includes the subjects of mechanics, heat, electricity, sound, and light.

The laws enunciated in physics are assumed, and the instruments described in physics are employed, in the other sciences.

Physics, then, is at the foundation of all the sciences. Physics itself is founded upon a body of observed facts which are interpreted by reason in accordance with the principles of mathematics. The exact expression of the relation between a large number of similar facts is called a *law*. Men had observed for ages that bodies unsupported in the air tend to fall toward the earth. It was only after many years of patient observation and the making of careful measurements that the fact became known that all bodies at the surface of the earth fall, if unimpeded, with the same velocity, the velocity gained being 32.16 feet (9.8 metres) for every second of time the body is falling. This fact, after being tested by innumerable experiments, is now known as the *law* of falling bodies.

Among the many new facts which are constantly being discovered there are always some, the relation of which to the body of known facts is not yet apparent. When a large number of such facts have been collected some bold philosopher offers an explanation, or as scientists say, a *hypothesis*. This hypothesis, if it seems to other scientists to be worthy of consideration, is tested to see if it will explain all the known facts. If it does explain the known facts, and especially if it leads to the discovery of new facts, it comes in time to be accepted as a *theory*. The Atomic Theory, for example, attempts to harmonize all the known facts of chemistry, and does so to a degree that was hardly anticipated by those who first proposed it. The theory seldom has so sure a foun-

dation as the separate laws or facts which it attempts to explain, yet it serves a most useful purpose, even when it does no more than aid us in keeping those laws and facts in memory.

2. **The Place of Physics in Education.** — The study of physics involves to an exceptional degree the training of the senses to perceive, the hand to perform, and the mind to interpret, on the one hand, what the senses observe; to direct, on the other, what the trained hand shall perform. In cultivating accuracy, it discourages slovenliness and untruthfulness and stimulates the mind to that highest of human pursuits, the search for truth.

3. **The Place of Physics in Practical Life.** — Mechanical inventions are not, as many thoughtless people suppose, mere lucky accidents; they are the direct result of a knowledge of the laws of nature. The man who discovers a new law of physics seldom sees how his discovery is to be of any practical benefit to mankind. Faraday, toiling patiently in his laboratory to discover the laws of electro-magnetic induction, never dreamed that his discovery of these laws would one day make possible the electric lighting of our homes and streets, the electric propulsion of railway trains, and the transmission of spoken words across a continent. Yet these are but a few of the applications of Faraday's discoveries to the daily life of men.

Enough has been said, it would seem, to make it evident that there are excellent reasons for giving physics a place in the course of study.

CHAPTER I.

MATTER AND MOTION.

4. Matter and Motion Defined. — We have seen already that physics is the study of the changes that occur in the things about us. We shall see as we proceed that all changes in the appearance of things are due to *motion*, either of the things themselves or of their parts.

Motion may be defined as change of position, the position of an object being determined by its distance and direction from other objects. In geometry we deal with motions of points and lines, as when a point is said to move so that its path is a straight line or a circle. All physical changes, however, imply the motion of *matter*.

Matter is a term with the meaning of which we are all more or less familiar, yet a term which it is not easy to define. We know matter primarily by the effort required to move it out of its place. If we walk in air we are so accustomed to the resistance which it offers to our movements that we think of our motion as unimpeded; but if we walk in water or through thick underbrush we are conscious of a resistance which can be overcome only by a considerable effort on our part. Matter also makes itself known to us through the senses

of sight, hearing, smell, and taste. It is chiefly through the senses of sight and hearing, especially the former, that we gain our knowledge of physical changes.

A definite portion of matter, separated from other matter, is called a *body*. The various kinds of matter of which bodies are composed are called *substances*. Thus a pencil is a body, composed of wood and graphite. Substances possess certain characteristic properties: cedar wood is of a reddish color, is brittle, and will float in water; graphite is black, leaves a trace on paper, and is heavier than water. The pencil possesses in its different parts the properties of the substances which compose it, and has besides a definite form and size.

The amount of matter in a body is called the *mass* of the body. The amount of matter in unit volume of a substance is called the *density* of the substance.

The various properties of bodies are made manifest to us by the motions which are transmitted from these bodies to our organs of sense. We shall best explain the properties of matter, therefore, by examining the motions of bodies. Meanwhile we must call to mind frequently many facts which have already come to our notice and keep ourselves ever alert to see new facts which may serve to illustrate the subject in hand.

5. Kinds of Motion. — I. A body may move from one position to another in such a way that the direction from one point in the body to another point in the body remains unchanged. Such motion is called motion of

translation, or *translatory* motion. Thus the cube *CDB* (Fig. 1) may have moved to the position *C'D'B'* in such a way that *CD* did not depart from the vertical direction. The motion was then translatory. If the successive positions of a point, as *C*, lie in a straight line, the motion is rectilinear; if in a curved line, curvilinear. Thus, if *C* followed the straight line *CFC'*, the motion was rectilinear; if the line *CEC'*, curvilinear; and if *CEC'* is a segment of a circle, it was circular.

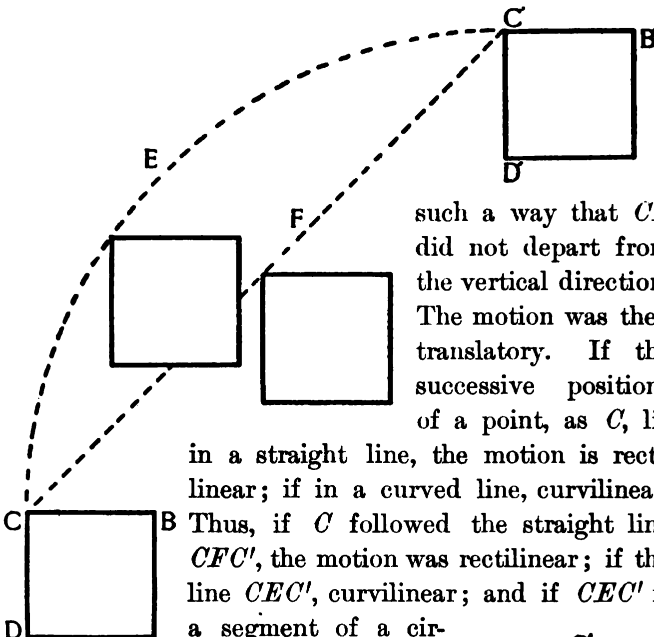


Fig. 1.

II. A body may move in such a way that the distance of every point in it from a fixed point remains unchanged, while the direction from one point in the body to another point in the body changes. Thus the cube in Fig. 2 may move from the position *CDB* to *C'D'B'*, the

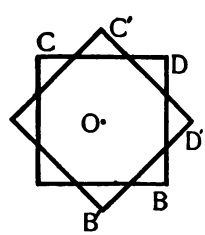


Fig. 2.

point C remaining at a fixed distance from O, while the direction of CD is constantly changing. Such motion is *rotary*. A line drawn through O such that all points on the line remain on the line during rotation is called the *axis* of rotation. The path of every particle in the body is a circle, having its centre on the axis.

III. A body may move in such a way that after moving a certain distance in one direction it stops and retraces its path in the opposite direction, as the pendulum of a clock does. Such motion is called motion of vibration, or *vibratory* motion. Vibratory motion is often so rapid that the eye cannot follow it. The prongs of a tuning fork, the string of a violin, indeed, every body which is producing sound vibrates thus rapidly.

IV. When a vibrating body is surrounded by a substance the particles of which may be set in vibration, the particles nearest the vibrating body will be set in vibration first. These particles will communicate their motion to those next them, and the disturbance will thus spread throughout the substance. Such motion is called *wave* motion. It is usually represented by a wavy line, as in Fig. 3.

FIG. 3.

The path of any particle is not such a wavy line, but is usually a short straight line or a small ellipse. No single particle moves forward, but each particle vibrates for a short time, sets its neighbor in vibration, and then comes to rest. The motion progresses from particle to particle,

each particle vibrating in turn. A water wave moves across the lake: a chip on the water moves up and down, but not forward. It is by wave motion that sound and light reach us from distant objects. The particles of the bodies themselves, and of the intervening medium as well, vibrate in very short paths; the sound or light — that is, the wave motion — travels to great distances.

6. The Cause of Motion, Force. — It is the universal experience of mankind that bodies not endowed with life have no power to put themselves in motion. Newton perceived also, what is equally true, that bodies once in motion have no power to stop themselves or change their direction of motion. If, then, we see a body which was at rest begin to move, or a body which was rising in the air begin to fall, we say something has acted upon it. Provisionally, while we are seeking to find what that something may be, it is convenient to give it a name. We call by the name *force* any cause which tends to change the motion of bodies. If a body, as a ship or a railway train, starts from the state of rest (the condition in which it has no motion relative to bodies near it), and moves due east, we say a force has acted upon it from the west. If it goes faster and faster toward the east we say a force continues to act from the west. If it goes more slowly eastward we say a force is acting from the east, which, if it continues to act, will bring the body to rest. If a stone is thrown horizontally it will not

continue to move in a horizontal path, but will gradually approach the ground. If we saw a boy throw the stone we say we know the cause of the horizontal motion. We saw nothing pushing or pulling the stone toward the ground, yet because it went downward we say there was a force acting in the downward direction. The fact that all bodies tend to move earthward has led us to give to that force which causes such motion toward the earth a special name, *gravity*.*

7. The Nature of Force. — As was suggested in the last paragraph, there are forces the nature of which is evident, and other forces the nature of which is not known. It may be said, however, that so far as we know the nature of force, every force is itself the result of motion. The wind is able to move the ship because the air is itself in motion. The steam in the cylinders of the locomotive is able to drive the train because motion in the form of heat was first imparted to the steam in the boiler.

The manner in which the motion of an engine when transmitted by belting to a dynamo-machine is so changed that it may be carried along a slender wire for miles to be again converted into visible motion in the street car is not yet well understood, but it seems probable that motion is present during every step of the process of transferring motion from the engine to the car, even though we are not able to perceive it.

* Gravity from Latin *gravis*, heavy, *gravitas*, weight.

So far as we know, every bird came from an egg, which egg, in turn, had a bird for its mother; so far as we know, every motion is the result of some antecedent motion.

8. Rate of Motion. Speed. Velocity. — We have all observed that bodies in motion do not all move equally fast. A strong wind will drive a windmill faster than a light breeze. A professional pitcher can throw a ball swifter than a schoolboy can. When distance travelled, only, and not direction of motion is considered, we call the distance travelled in one second the *speed* of a body. *Velocity* is rate of change of position, and includes both speed and direction of motion. If the distinction between speed and velocity is carefully made much confusion will be avoided. A train which travels 240 miles in four hours has an average speed of 60 miles per hour, or one mile per minute, or 88 feet per second.

$$Speed = \frac{length\ of\ path\ traversed}{time\ occupied},\ \text{or}$$

$$(1) \quad s = \frac{l}{t}$$

9. Change of Velocity. Acceleration. — Motion with constant speed and in a constant direction is constant motion. Motion which changes either in speed or direction is called accelerated motion, and the rate of

change of motion of a body is called its *acceleration*. A body which moves five centimetres in the first second, ten centimetres in the second, fifteen in the third, etc., has a constant acceleration of five centimetres per second. A body moving in a circle with uniform speed is an example of accelerated motion; for, though the speed remains constant, the direction of motion is constantly changing. It takes a force constantly acting to produce a constant change of speed in the first case. It takes a force constantly acting to produce the constant change of direction in the second case. If a heavy body be whirled by a string held in the hand we are conscious of a constant pull which we must exert upon the string to keep the ball from moving off in a straight line.

We may now define force as any cause capable of producing acceleration.

To sum up:

$$\textit{Acceleration} = \frac{\textit{change of velocity}}{\textit{time in which change occurred}}, \text{ or}$$

$$(2) \quad a = \frac{v}{t}$$

10. Quantity of Motion. Mass. Momentum. — A given force will produce a greater change of velocity in a light body than in a heavy one. The amount of acceleration alone is not a measure of the magnitude of a force: we must also take into account the amount of matter moved. The quantity of matter composing

a body is called the *mass* of that body. The mass of a body is measured by the acceleration a given force will produce upon the body. The greater the mass the smaller the acceleration produced by a given force. We are accustomed to form a rough estimate of the force we ourselves exert by the effort we are conscious of putting forth. We are conscious of more effort when we push aside a brick with the foot than when we push aside a block of wood of the same size. We say the brick is harder to move than the block, and therefore has more matter in it.

Quantity of motion is called *momentum.* It is equal in amount to the product of the numbers representing the mass and velocity of the body.

$$\textit{Momentum} = \textit{mass} \times \textit{velocity}, \text{ or}$$
$$(3) \quad M = mv$$

Since the change of momentum of a body is the measure of the force applied to it, and since the mass of the body is not supposed, in our discussion of motion, to suffer any change in amount, it follows that the force expended in producing any given motion is measured by the product of the number of units of mass times the number of units of acceleration.

$$\textit{Force} = \textit{mass} \times \textit{acceleration}, \text{ or}$$
$$(4) \quad f = ma$$

With these preliminary definitions in mind, we should now be prepared to state and discuss the three famous

axioms of Sir Isaac Newton, which are known as Newton's Laws of Motion.

11. Newton's Laws of Motion. — I. Every body continues in its state of rest or uniform motion in a straight line unless acted upon by some external force.

II. Amount of motion is proportional to the force applied and in the direction in which the force acts.

III. Action and reaction are equal in amount and opposite in direction.

12. Discussion of the First Law. — We have seen in Section 6 that force is required to change motion, that is, to alter the speed or direction of a body. If the body is at rest its speed is zero, and, since it has no motion, it has no direction of motion. If no force is applied the body remains at rest.

When a body is in motion a force would be required to produce any acceleration of that motion, that is, to change either the speed or direction of the motion. It follows that the body continues in its present state unless a force is applied. The helplessness of matter is called *inertia*, and force is sometimes said to be spent in overcoming inertia. Since, however, inertia is a purely negative property of matter it seems better to say that force is spent in producing change of motion than to say that force overcomes the tendency of bodies not to move.

Illustrations of the first law will occur to every one.

A person who jumps from a moving car keeps the motion of the car till his feet strike the ground. His head continues to move forward even then, and unless he runs forward it is quite probable that his head, too, will strike the ground before it loses all its motion. Drops of water fly off from a rapidly rotating grindstone. Bits of mud are thrown from a carriage wheel. Railways have the outer rail laid higher than the inner where sharp curves occur to prevent the engine from leaving the track, as it would do if it kept on in a straight line. The rotation of the earth causes the water of the ocean to heap up at the equator until it is about thirteen miles higher there than at the poles. The Mississippi River, therefore, notwithstanding its source is 1,575 feet above sea level, is really about two miles higher at its mouth than at its source. If the earth should cease to rotate the Mississippi would flow rather swiftly toward the north.

13. Discussion of the Second Law. Composition and Resolution of Forces. — In Section 10 we have defined "amount of motion," or momentum, as mass times velocity. The fact that the velocity of a body has changed is proof that a force has acted upon it. The second law implies that no matter how many forces are acting upon a given body at one time each force will produce the same effect as if it alone were acting, and the total effect must be found by combining the effects of all the forces. This may be done in a very

simple way when the magnitude and direction of each of the forces are known. A line has both length and direction. A line may be used, therefore, to represent the magnitude and direction of a force. Let the magnitude of each force be represented by the length, and the direction of each force by the direction, of a line. The absolute length of the lines is of no importance so long as their relative lengths correspond to the relative magnitude of the forces. Suppose it is required to find the magnitude and direction of the force which is the equivalent of two forces, one a force of magnitude, 4, acting from the west, the other a force of magnitude, 3, acting from the south, as if A should kick a foot-ball from the south with a force of three pounds, while at the same instant B kicks it from the west with a force of four pounds.

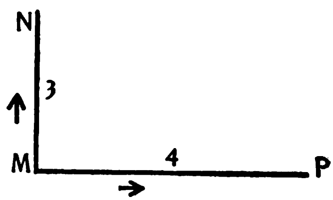


FIG. 4.

Represent the force of 3 from the south by the line *MN* (Fig. 4) and the force of 4 from the west by the line *MP*. Both forces are supposed to be applied at *M*. The arrowheads indicate the direction of motion.

Now, since each force produces the same effect as if it alone were acting, we may suppose the ball to move long enough under the influence of the first force alone to reach the point *N*, then under the influence of the second force only to move toward the west from *N* for a like length of time. The ball would arrive in the

time specified at the point R, which is the same point it would reach if the two motions occurred at the same instant. The ball has now moved from M to R, hence the line MR represents by its length and direction the magnitude and direction of a force which is equivalent to the two forces MN and MP acting at M. This equivalent force is called the resultant of the other two forces. Its numerical value is readily found from geometry:

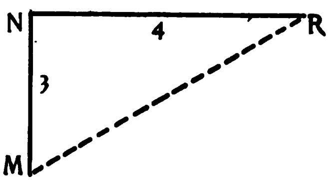


FIG. 5.

$$\overline{MR}^2 = \overline{MN}^2 + \overline{NR}^2 = 25 \qquad \overline{MR} = 5$$

When the directions of the forces make any other than a right angle with each other the value of the resultant may be found by constructing a good-sized diagram to scale and measuring the length of the line which represents the resultant.

If more than two forces are acting at a point the lines representing the forces are joined end to end. The line connecting the beginning and end of the chain thus formed represents the resultant. Thus

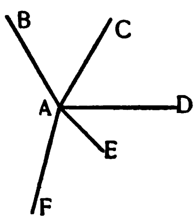


FIG. 6.

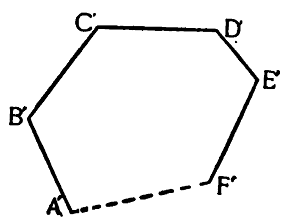


FIG. 7.

forces AB, AC, AD, AE, AF, act at the point A (Fig. 6).

Draw
$$\begin{aligned} A'B' &\parallel AB \\ B'C' &\parallel AC \\ C'D' &\parallel AD \\ D'E' &\parallel AE \\ E'F' &\parallel AF \end{aligned}$$

Then $A'F'$ (Fig. 7) is the resultant.

The method employed for two forces is commonly called the *triangle* of forces. It is but one case of the more general method called the *polygon* of forces. If the two forces act in the same straight line their resultant is evidently their sum or their difference, according to whether they act in the same or in opposite directions.

The process of finding the resultant of two or more forces is called the *composition* of forces.

Since the velocity produced upon a given mass is proportional to the force producing it, it is evident that velocities may be compounded exactly as forces are.

It is often desirable to resolve the force acting upon a body into two or more forces. The process is called *resolution* of forces. It is the converse of composition of forces. A common case is that of a body moving along a path which is not parallel to the direction of the force acting, as when a ship sails westward by means of a south wind. A somewhat simpler case is the following: A car standing upon track 1 (Fig. 8) is to be pushed west by a switch engine on track 2 by means of a rigid iron bar AB. According to the second

law of motion, the force which moves the car west must act in that direction. Only a portion of the force *AB* acts westward. Let us resolve *AB*, then, into two components, one westward, *CB*, producing motion, one northward, *AC*, producing pressure on the tracks.

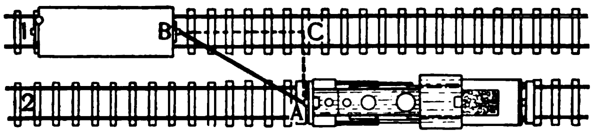


FIG. 8.

Knowing the distance apart of the tracks and the length of the bar *AB* we can at once determine the numerical value of each of the two components.

The reason is now obvious why the resultant of several forces not in the same straight line is always less than the sum of the forces. It is also evident that when the direction of motion is not in the direction of a certain force there must be other forces acting at the same time with that force. The direction of motion is always the direction of the resultant of all the forces acting.

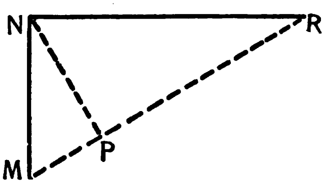


FIG. 9.

In the case given on page 16 we may resolve *MN* (Fig. 9) into two forces, *MP* producing motion and *PN* not producing motion. Similarly we may resolve *NR* into *PR* producing motion and *NP* not producing motion. The sum of the forces producing motion is:

$MP + PR = MR$. The sum of the forces not producing motion is: $NP + PN = 0$.

14. Discussion of the Third Law. — The third law of motion implies that there are always two bodies concerned in every motion. If a moving bat strikes a ball which is moving in the opposite direction the amount of motion destroyed in each is equal. If a croquet ball at rest is struck by a mallet so that the mallet comes to rest the ball will move forward with a momentum equal to that which the mallet had before impact. The ball has exerted as much force in stopping the mallet as the mallet has exerted in moving the ball.

If a man in a large boat draw a small boat toward him in the water by means of a rope both boats will move, but the small one will move faster, and, therefore, farther, than the large one. Their relative velocities may readily be found as follows: Call * m_1, v_1, M_1 and m_2, v_2, M_2 the masses, velocities, and momenta of the two boats.

$$\text{Then} \quad M_1 = m_1 v_1$$
$$M_2 = m_2 v_2$$
$$\text{and since} \quad M_1 = M_2$$
$$m_1 v_1 = m_2 v_2$$
$$\text{If we suppose} \quad m_1 = 5$$
$$m_2 = 2$$
$$v_1 = 10$$
$$\text{then } v_2 = \frac{2 \times 10}{5} = 4$$

or the small boat will move two and one half times as fast as the large one.

* Read m_1 "m sub one."

MEASUREMENT. UNITS.

15. The Fundamental Units. — In our discussion of motions and the forces which produce them we have assumed that it is possible to express in numbers the mass of a body, the distance it travels, and the time spent in motion. The exact determination of these and similar quantities is one of the principal tasks of the physicist, for upon such measurements every law and theory in physics rests.

A moment's reflection will show that to measure velocity we must measure distance and time, to measure momentum we must measure mass, distance, and time. It has been found possible to express all physical quantities, electrical and magnetic included, in terms of the three fundamentals, *length*, *mass*, and *time*.

For the measurement of any quantity we require a unit of quantity of the same sort, the magnitude of which has been determined by comparison with some standard agreed upon by scientists. The system of units based upon the metre and known as the metric system is in universal use by scientists and is fast being adopted by the civilized nations of the world for use in commercial transactions.

A *metre* is the length of a certain bar preserved at Paris, copies of which are in the possession of the Bureau of Weights and Measures at Washington. Its length is equal to 39.37 English inches, or about 1.1 yards. The metre is divided into 100 centimetres, as our dollar is divided into 100 cents. The centimetre is

divided into ten millimetres. For long distances the kilometre is used. A kilometre equals 1,000 metres. Tables of English equivalents will be found in Part II.

The unit of mass is the *gram*. A gram is the mass of one cubic centimetre of water at a certain temperature (39.8° F. = 4° C.). The gram is divided into centigrams and milligrams, the prefixes centi and milli denoting here, as always, hundredths and thousandths of the unit. A mass of 1,000 grams is called a kilogram. It is equivalent to 2.2 English pounds. A new five-cent nickel weighs five grams. A litre of water (1,000 cubic centimetres) weighs, of course, one kilogram.

The unit of time used by physicists is that which we use in daily life, the *second*. It is $\frac{1}{86400}$ of a mean solar day.

Methods of measuring these fundamental quantities are explained in Part II of this book.

The system of units based upon the centimetre, gram, and second is known as the c. g. s. system.

16. Derived Units. — The unit of velocity has received no special name. A body has unit velocity when it moves unit space in unit time, hence in the c. g. s. system the unit of velocity is the *centimetre-per-second*. The unit of momentum is the *gram-centimetre-per-second*. The unit of acceleration is the *centimetre-per-second-per-second*. The unit of force is the *gram-centimetre-per-second-per-second*. Happily it has received

a shorter name, the *dyne.** A dyne is the force which, acting for one second upon a mass of one gram, will accelerate its velocity one centimetre per second. For example, a body is acted upon for one second by the force of gravity. It is required to find the intensity of the force which gravity exerts upon it. It has been found that, no matter what the mass of the body may be, it will gain in one second a velocity of 980 cm. The force acting upon each gram of the body is therefore 980 dynes.† Forces of considerable size may be conveniently measured in grams or even in kilograms. The value may then be expressed in dynes, if the exact value for the force of gravity per gram of mass is known at the place where the measurement is made.

The exercises which follow are intended to illustrate and fix in mind the principles treated in this chapter. For convenience the various formulæ are collected and placed at the head of the exercises. They should be fully understood and thoroughly memorized. We shall meet them again and again.

Formulæ.

(*1*) $s = \frac{l}{t}$ *whence* (*1a*) $l = st$ (*1b*) $t = \frac{l}{s}$.

(*2*) $a = \frac{v}{t}$ *whence* (*2a*) $v = at$ (*2b*) $t = \frac{v}{a}$

(*3*) $M = mv$ *whence* (*3a*) $m = \frac{M}{v}$ (*3b*) $v = \frac{M}{m}$

(*4*) $f = ma$ *whence* (*4a*) $m = \frac{f}{a}$ (*4b*) $a = \frac{f}{m}$

* Dyne from Greek *dyn*, force.

† This value is approximate. It varies in reality from 977 to 984 at different places on the earth's surface.

Exercises.

1. What kinds of motion are illustrated (*a*) in the sewing machine; (*b*) in the bicycle?

2. When a top is set spinning (*a*) what keeps it spinning; (*b*) what makes it finally stop? What law of motion is illustrated?

3. An ice boat has sharp runners, so that it cannot slip sidewise. It is directed toward the northeast. If the wind is blowing ten miles an hour from the south is it possible for the boat to be driven by it faster that ten miles an hour toward the northeast? Explain.

4. A horse pulls a loaded wagon exerting a force of 300 pounds. (*a*) How many dynes is that? (*b*) Where is the 300 pounds of reaction required by the third law of motion? (*c*) If he were on smooth ice with perfectly smooth shoes, could he do it?

5. The earth rotates once in twenty-four hours. Calling its circumference 25,000 miles, what is the speed of a point on the equator (*a*) in miles per hour; (*b*) in kilometres per hour; (*c*) in feet per second; (*d*) in metres per second? (*e*) Why does it not stop, as the top does?

6. (*a*) Which law of motion is illustrated by the "kick" of a gun? (*b*) Why does not the gun hurt as badly as the bullet does?

7. A body starts from rest and gains constantly in velocity until at the end of ten seconds it has a speed of 20 metres per second: (*a*) What was its acceleration? (*b*) How far did it travel?

Solution.

(*a*) by equation (*2*) $a = \frac{v}{t} = \frac{20}{10} = 2$ metres per sec.

(*b*) by equation (*1*) $l = st = \frac{0+20}{2} \times 10 = 100$ metres.

NOTE. — The speed s must be taken as the *average* speed, which is, of course, equal to half the sum of the initial and final speeds when the acceleration is constant.

8. A ball rolls down a smooth inclined plane 10 metres long and 6 metres high (see Fig. 10). We may resolve the weight of the ball into two components one $\parallel$ and one $\perp$ the plane, that is, one in the direction of motion and one at right angles to it. According to the second law of motion the first component only produces motion. The total force of gravity upon the ball would produce an acceleration of 9.8 metres per second per second. What acceleration will the component *ca* produce?

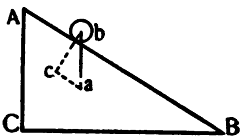


FIG. 10.

NOTE. — Since the right triangles ABC and abc have the sides $AB \perp bc$ and $ab \perp BC$ they are similar, and $\frac{ca}{ba} = \frac{CA}{BA}$.

9. A body falls freely under the influence of gravity. It has a constant acceleration of 9.8 metres per second per second: (*a*) What is its velocity at the end of five seconds? (*b*) How far has it fallen?

10. A man who can lift but 120 pounds wishes to lift a 200-pound barrel of salt into a wagon 3 feet high. If he is to roll it up a plank, how long a plank must he use?

11. A car weighing 1,000 kilograms moves east with a velocity of 20 metres per second, and strikes a car weighing 600 kilograms which is at rest. Both cars now move east: (*a*) What is their joint momentum; (*b*) their joint velocity after impact?

12. A Spanish ball weighing 8 kilograms and moving north at the rate of 50 metres a second meets an American ball weighing 20 kilograms and moving south at the rate of 20 metres per second: What is the velocity of the combined mass after impact?

CHAPTER II.

BALANCING FORCES.

17. We are all of us compelled after deliberation to accept the truth of Newton's first law of motion, and yet our own observation is that the bodies we see moving do not continue in motion long after the force which sets them going ceases to act. The condition of comparative rest rather than a state of motion seems to us the normal state of inanimate objects. To see a moving body come to rest does not surprise us. It is when a body continues long in motion without the visible application of force that our wonder is aroused.

The explanation of this seeming contradiction is found in the fact that moving bodies impart motion to all bodies which they touch and that no body can move at the surface of the earth without touching something. A body moving in the air must push the air to one side, it must put the air in motion. It will lose its own motion, therefore, little by little till it will ultimately come to rest. But the air is not the chief hindrance to motion in most cases. Gravity is constantly acting. The body falls to the earth. If it rolls along uneven ground it will be lifted out of the hollows by its own momentum until, in rubbing against the earth, it has lost so much of its motion that it finally comes to rest in a

hollow, out of which it has not sufficient momentum to lift itself (see Fig. 11). This rubbing of one body against another is known as *friction.* It is in reality of the same nature as the hindrance due to the large inequalities of the surface of the ground just mentioned. The smoothest surface, when examined under the microscope, is seen to be covered with little hollows and projections. When one body rests upon another the projections of the one fit into the hollows of the other (see Fig. 12). Before the first body can slide along the surface of the second it must be lifted enough to let the projections of the two bodies clear each other.

FIG. 11.

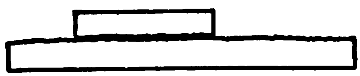


FIG. 12.

While the motion continues there is still a tendency for the projections to drop into the depressions, hence friction continues until the moving body comes to rest. Polishing the surfaces reduces the height of the projections and so diminishes friction. The application of oil or graphite fills the depressions and has a like effect. The heavier the moving body the greater the friction. This indicates that the force which brings the body to rest is gravity. Now, gravity does not cease to act when the body comes to rest, as you may convince yourself by holding a heavy weight in your hand while the hand rests upon the table. In fact, the condition of rest is never proof that no force is acting upon the

body which is at rest; it is proof that the forces acting are *balanced*, that is, that the resultant of all the forces acting upon the body is zero.

This balanced condition has received the name *equilibrium*. When the forces acting upon a body are in equilibrium any one of the forces is equal in magnitude and opposite in direction to the resultant of the remaining forces.

18. Kinds of Equilibrium. — A block lying upon the ground in any one of the first three positions shown in Fig. 13 is in equilibrium with reference to gravity.

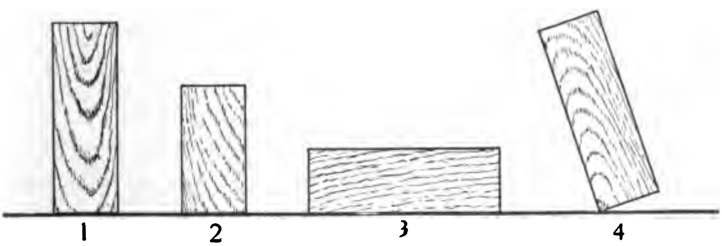


FIG. 13.

It is evidently more easily overturned in 1 than in 2, and less easily in 3 than in 2 or 1. It might be balanced also in position 4, but in that position it would be overturned by the slightest application of force in almost any direction. In 1, 2, or 3 the body must be lifted to overturn it. In 4 the body is as high as it can be and touch the ground. The equilibrium in the first three cases is said to be stable, in the last case

unstable. The different degrees of stability in the first three cases may be more clearly shown if we first introduce the idea of centre of mass. Let a body be supported by a string attached to it at any point, and let the body come to rest. If we imagine a plane drawn from the point of support downward toward the centre of the earth the body will be divided into two parts, such that the force of gravity acting upon the matter in one part of the body exactly balances the force acting upon the matter in the other part of the body as far as producing rotation about the point of support, *F* (Fig. 14), is concerned. Let a second plane of the same sort be drawn through *F*. Now support the body at a new point, *F′*, and draw a vertical plane through that point also. The point of intersection of the three planes is the centre of mass of the body.

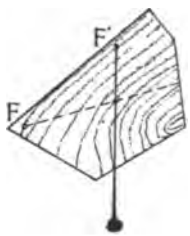


FIG. 14.

A force directed through the centre of mass has no tendency to set the body in rotation. A force applied to the body, not through the centre of mass, does tend to produce rotation. When a body is supported at a point directly under or over its centre of mass, gravity does not tend to make it rotate. We say, therefore, that gravity acts upon a body exactly the same as it would if the mass of the body were collected at the centre of mass. The centre of mass is often called the centre of gravity. When a body as a whole is lowered

or raised, that is the same as lowering or raising its centre of mass. In Fig. 15 let C, C' represent in each case the positions of the centre of gravity as the body is overturned. It is evident that the distance the centre

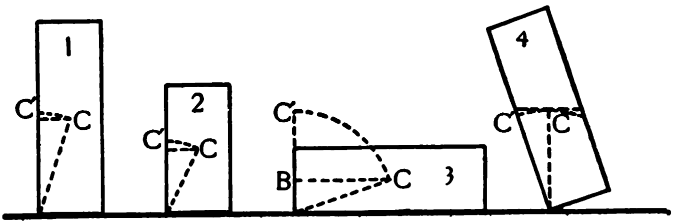


FIG. 15.

of mass is lifted is greatest in 3, which is, therefore, the position of greatest stability. In 4 the least motion will lower the centre of mass, and hence the equilibrium is unstable.

The sphere (1, Fig. 16) is neither stable nor unstable, since its centre of mass is neither raised nor lowered by overturning it. The equilibrium is, therefore, said to be *neutral*.

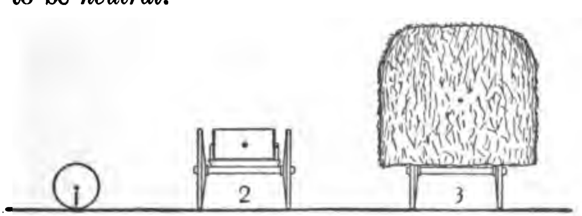


FIG. 16.

The conditions for stability are that the centre of mass should be low and that the base, that is the

figure formed by connecting the points of support, should be large. The load of hay (3, Fig. 16) is less stable than the load of stone (2, Fig. 16), because its centre of mass is higher. It would be more likely to upset, therefore, on a rough road, than would the load of stone.

19. Moments.—If a rigid bar be supported near its centre of mass it will be in neutral equilibrium. Experiment shows that two equal forces acting downward upon the two sides or arms of the lever will not produce equilibrium, except the forces be applied at equal distances from the centre of rotation, and that a force of 1 at a distance 2 will balance a force of 2 at a distance 1.

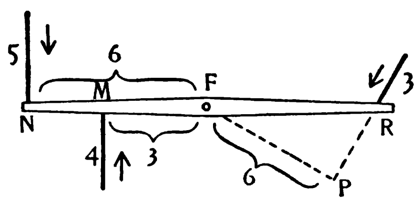


FIG. 17.

In general the effect of a force in producing rotation is measured by the product of the magnitude of the force, times the distance of the line of application of the force from the centre of rotation. This product is called the *moment* of a force.

Moment of a force = force × distance from centre.

The distance to the line of application of the force is, of course, the perpendicular distance.

The condition of equilibrium for a lever, as for any body free to rotate about a fixed axis, is that the sum of the moments of all the forces acting upon it shall be zero.

In the case illustrated in Fig. 17 the sum of the moments of the forces M and R tending to rotate the lever toward the right is: $4 \times 3 + 3 \times 6 = 30$. The moment of the force N is $5 \times 6 = 30$ toward the left. Since the sum of these moments is zero the lever is in equilibrium.

20. The Lever Balance.—The force of gravity at any place is the same for equal masses. A lever suspended at a point slightly above its centre of gravity has attached at equal distances from its centre pans in which the masses to be compared are placed. The lever, or beam as it is called, should be horizontal when the pans are empty.

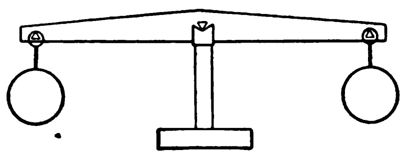


FIG. 18.

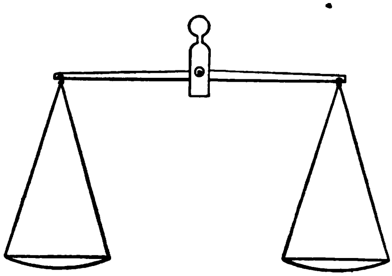


FIG. 19.

It will again be horizontal when the masses in the two pans are equal (see Figs. 18, 19).

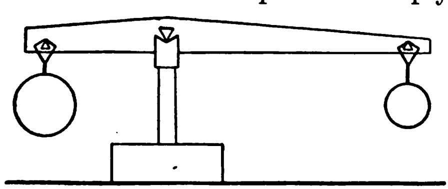


FIG. 20.

When large masses are to be compared with smaller ones a lever with unequal arms (Fig. 20) is employed. A

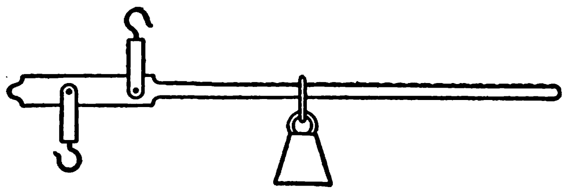


FIG. 21.

familiar form is called the steelyards (Fig. 21). In the large scales used for weighing grain or coal combinations of several levers are employed, so as to reduce the size of the standard weights used.

21. Universal Gravitation.—With truly prophetic insight, Sir Isaac Newton conceived that the force of gravity, which acts between the earth and all bodies upon its surface, reaches also to the moon and to all the planets of our solar system—indeed to the limits of our visible universe. Certain it is that some force must be constantly acting to keep the moon constantly changing its direction as it revolves about the earth in a nearly circular orbit. Certain it is, too, that gravitation (of which gravity is but a special example), acting in accordance with the laws enunciated by Newton, is fully competent to keep the planets in equilibrium, each maintaining its average distance from the sun constant from century to century.

22. Newton's Law of Gravitation. — Every particle of matter in the universe attracts every other particle. The force impelling any two bodies toward each other is proportional to the product of the masses of the two bodies and inversely proportional to the square of the distance between their centres of mass.

If A, B are two spheres whose masses are m_1, m_2, and distance between centres, r, then:

$$(5) \quad f = \frac{K\, m_1\, m_2}{r^2}$$

where K is a number which is constant for any set of units used in measuring m and r. It is the attraction between two unit masses (1 g. each) whose centres of mass are unit distance (1 cm.) apart. In dynes $K =$ .000000065. It is a very small quantity, as one can readily see, since the force between the earth and a body weighing a gram is only 980 dynes, while the mass of the earth is so great as it is.

The force between two kilogram weights at a distance of ten centimetres is so small that very delicate instruments are required to detect it at all.

The force between the earth and the moon is much greater. If the moon could be stopped in its revolution about the earth it would fall at once to the earth. As it is, the force of gravitation is employed in deflecting the moon's motion from a straight line. Gravitation, then, tends to equilibrium. In the case of bodies moving in orbits a sort of equilibrium — an equilibrium of motion, in which the *average* distance of a body from

neighboring bodies remains constant — has been already attained. All bodies which are actually falling toward each other are tending to a condition of equilibrium. At the surface of the earth bodies must first be lifted before they can fall. When a body is once lifted we know the law, we learned it in childhood: "All that goes up must come down."

SOME PROPERTIES OF MATTER.

23. Elasticity. — A body was defined in Section 4 as a portion of matter having a definite size and shape. Now the size and shape of a body may be changed somewhat by the application of force without *permanently* altering either its size or shape. As soon as the force ceases to act the body returns to its original condition. This could only be true if a force acted to make it so return. We call the forces concerned in preserving constant the size and shape of bodies elastic forces, and the property which different substances possess in different degrees of exhibiting elastic force we call *elasticity*.

Any force which acts upon a body so as to change its form or size from that which it would assume under the action of the elastic forces is called a ***stress***. The effect of a stress in deforming the body is called a ***strain***. When the stress is removed elasticity causes the body to recover from the strain and return to its original form and size.

If a metre stick, with its ends resting upon two blocks or books, have a kilogram weight placed upon it at its middle, it will be depressed until the elastic force of the wood is equal to one kilogram, when it will come to rest. If the weight be now removed the stick will again become straight.

The air in a boy's pop-gun is compressed against an elastic force which increases until the wad is forced out, when the air returns at once to its original volume.

24. Pores, Molecules, Cohesion. — In order to understand how the same body can occupy a different amount of space at different times we must suppose that before the air in the pop-gun, for example, was compressed, there were portions of space in it which were not occupied, so that some of the particles could be forced into these pores. There are excellent reasons for thinking that all bodies are composed of very small particles which have been called molecules.* The pores are the spaces between the molecules. Two forces are constantly acting, one to draw the molecules toward each other, one to push them apart. The force which draws the molecules together is *cohesion* (called adhesion when it acts between two different bodies). In what respects cohesion differs from gravitation is not yet known. It is effective only through very small distances. Very fine iron dust will remain dust

* Mol′e-cule, Latin *moleculum*, from *moles*, mass, and *culum*, little — a little mass.

though its particles are in contact. Under heavy pressure, however, most powdered substances will cohere, especially if they are first moistened or heated, but the structure of the body thus formed differs from the structure of the substance before it was ground to powder.

The force which balances cohesion is called *heat.* It is a vibratory motion of the molecules which prevents their coming closer together. If a body is made hotter, therefore, its molecules are driven farther apart — it becomes larger. If the body loses heat it grows smaller, until cohesion and heat exactly balance each other. In a *solid* body cohesion so far predominates over heat that the molecules retain certain definite positions with reference to each other. If enough vibratory motion is imparted to the molecules in the form of heat, cohesion will be so far overcome that it will no longer be able to keep the body in a definite shape, the substance becomes *liquid* and under the influence of gravity takes the shape of the vessel in which it is contained, its upper surface being nearly horizontal. If heat be still further applied the liquid will expand for a time, but if a sufficient amount of heat is imparted to the body the particles will be driven so far apart that cohesion loses all control and the body becomes a *gas.*

Most substances may exist in any one of these three forms. All gases are now liquefied in the laboratory. All solid bodies may now be volatilized, or turned to gas. Some substances do not readily take the liquid

form but pass at once from solid to gaseous, or from gaseous to solid. This matter will be discussed more fully under the subject of heat.

In Fig. 22 let the small black dots at *a* represent the position of the molecules of a small body at a given instant. The circles may represent the range of the motion of the individual molecules. At *b* more heat has been added to the body, the molecules require more room in which to vibrate, the body is larger. At *c* after more heat has been added the body has become a liquid. To represent on the same scale the gaseous condition would require a diagram too large for this book.

FIG. 22.

25. **Elasticity Explained.** — We may explain in the light of the statements just made the phenomena which we have ascribed to elastic forces. In short, heat and cohesion are the elastic forces. In Fig. 23 let *a* represent a bar of steel. Let *b* represent the same bar, which has been bent by applied forces into the form shown. The molecules in the lower row are nearer together than they normally are, those in the upper row farther apart. Heat is striving to drive the lower row apart, cohesion tends

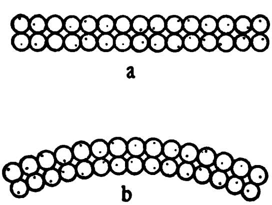


FIG. 23.

to draw the upper row together. In other words, the body tends to return to its original shape. Elasticity of shape is thus seen to be exactly like elasticity of size. In both cases it is the effect of forces tending to prevent expansion or compression. In solids different parts of a body may be compressed or expanded unequally. In liquids and gases this is impossible, hence liquids and gases (both of which are called fluids*) have no form and exhibit only elasticity of size or volume. Owing to the freedom of motion of the molecules in fluids, all fluids are highly elastic.

26. Liquids in Open Vessels. — A liquid in an open vessel is acted upon not only by the elastic forces but by gravity. Every particle of the liquid is pressed downward by the weight of the liquid above it and by the weight of the air. Let *a* (Fig. 24) be a molecule of water. It is pressed downward by the weight of the air and the water above it. But since it is at rest it must be pressed upward by an equal amount. Moreover, if it were not pressed from all sides by a like amount it would slip out of its place, since the particles of a liquid are free to move among themselves. The fact is that the molecules *c*, *d*, and all others which are the same

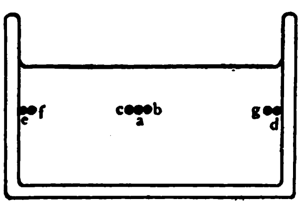


FIG. 24.

* Latin *fluere*, to flow.

distance below the surface are pressed downward with the same force that *a* is. They must all press sidewise by a like amount or the liquid would not be at rest. The molecules at *e*, *d*, press the sides of the vessel with a force equal to that exerted upon them by their neighbors *f*, *g*. Any surface on the line *de* has a pressure upon it which is proportional to the area of the surface, but is the same for all equal surfaces for that depth and is alike in every direction.

Let us now suppose that by tilting the vessel suddenly the water is thrown into the position shown in Fig. 25. The pressure upon *c* is less than that upon *b*, consequently *a* will be pushed away from *b* with a greater force than it is pushed away from *c*. The same is true of other particles, hence they will move to the left till equilibrium is restored, that is, until all parts of the liquid surface are at right angles to the direction of gravity. The last statement presupposes that the body of liquid is so large that gravity is the principal force acting.

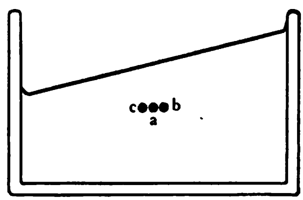


FIG. 25.

If other forces are acting the liquid will come to rest with every point of its surface perpendicular to the resultant of all the forces acting at that point.

Let us first consider the case of a liquid acted upon by gravity only. It follows from the principle just

stated that in any number of communicating vessels, no matter what their size or shape, a liquid will have the same level (see Fig. 26). In a tea-pot the liquid rises

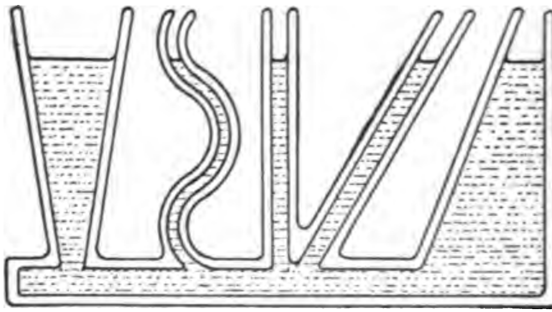


FIG. 26.

as high in the spout as in the pot itself and no higher. The pressure of any particle at *a* (Fig. 27) due to the liquid in *C* must be exactly equal to that due to the liquid in *D*, or the liquid would not come to rest. If the height of the liquid above *a* were greater in *D* than in *C*, the pressure would not be equal and there would be a movement from *D* to *C*.

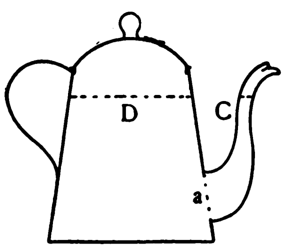


FIG. 27.

We may now sum up the facts in regard to the behavior of liquids in open vessels in the following statements:

I. The pressure at any point in a liquid is the same in all directions.

II. The pressure at any point in a liquid of uniform density, due to the weight of the liquid, is proportional to the depth of the point below the surface of the liquid.

III. The surface of a liquid at rest is perpendicular at any point to the resultant of all the forces acting at that point.

If we now examine some cases in which gravity is not the greatest force acting upon the liquid, we shall see that they are apparently, but not really, exceptions to the third law.

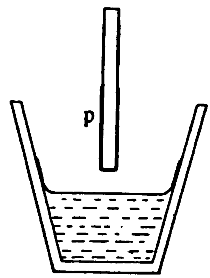


FIG. 28.

27. Capillary Phenomena. — If a piece of clean glass be dipped in water some of the water will adhere to the glass when it is removed (Fig. 28). Here adhesion is more powerful than gravity, and the surface of the water may be almost parallel to the direction of gravity (Fig. 28). Let us again dip the plate of glass in water (Fig. 29). The water will be drawn to the water on the glass by cohesion, to the glass by adhesion, and downward by gravity. Very near the glass adhesion is most powerful, hence the resultant is nearly horizontal, as at P (Fig. 30); a little

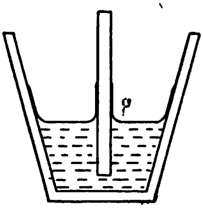


FIG. 29.

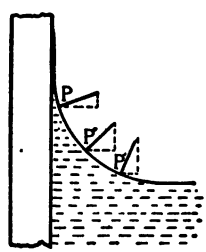


FIG. 30.

distance from the glass gravity is much the greatest force acting, while at P″ the forces are nearly equal in magnitude.

If two glass plates are placed very near each other in water the weight of the liquid between them is small as compared with the adhesion between the water and the glass, hence the water rises to a considerable height above the level of the rest of the water in the glass. In a very fine glass tube the weight of water is exceedingly small as compared to the surface of glass and water in contact. In such tubes water rises to a height of several centimetres (*a*, Fig. 31). Such tubes are called capillary tubes,* and the phenomenon of elevation of a liquid in capillary tubes is called *capillarity*.

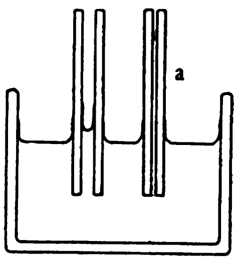


FIG. 31.

If instead of clean glass we had used oily glass, to which water does not adhere, or if instead of water we had used mercury, cohesion in the liquid itself would have caused a depression of the surface where the liquid touched the glass, as shown in Fig. 32.

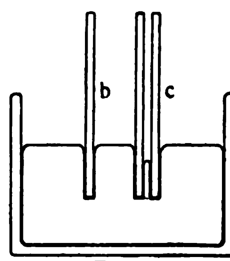


FIG. 32.

* Latin *capillus*, a hair.

28. Surface Tension. — Water falls through the air in drops. Water on the surface of the leaves of plants to which it does not adhere collects in spheroidal masses. Mercury spilled upon the table collects in spheroids, the smallest of which are nearly perfect spheres, the larger being flattened by their greater weight (Fig. 33).

FIG. 33.

A glass rod or a stick of sealing wax, when broken, shows sharp edges. If the broken end of the glass rod or the stick of wax be held in a flame to soften it the sharp corners are rounded off. How can these phenomena be explained?

The liquids in question (for the wax and glass are melted in the flame) behave as if their surfaces were stretched, or, in scientific language, under tension. The drop of water has least surface when it is a perfect sphere. The mercury in Fig. 32 is pushed down by the glass plate, just as if there were a skin of rubber over the surface. For this reason all capillary phenomena, as well as the other phenomena just referred to, are usually explained as results of surface tensions. It remains to be explained why liquids are under tension at their surfaces.

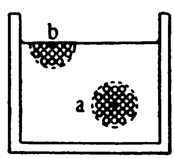


FIG. 34.

A molecule of water at *a* (Fig. 34) is acted upon by the cohesion of all of the molecules within a certain small distance from it, but it is drawn equally in all directions and is, therefore, in equilibrium. A molecule near the surface,

as *b* (Fig. 34), is acted upon only from one side, and so there is always a force acting inward. It is transmitted, of course, throughout the liquid and produces the same effect as would a thin elastic rubber bag stretched over the liquid. We know that if we push in a stretched membrane at one point, it tends when released to come back to a flat surface again. A drop of water or of any liquid behaves in the same way. The surface of any liquid will tend to take the shape that will make it the smallest possible. The tension is along the surface, hence when the surface is flat there is no tendency to

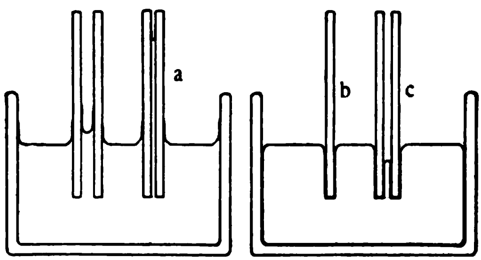


FIG. 35.

motion. If, however, the surface is curved, as when a tube is pushed down into a vessel of mercury (*c*, Fig. 35), the tension of the surface tries to shorten the surface and so pulls the mercury downward. In case of a wet glass tube in water (*a*, Fig. 35), the surface is shortened if the water rises. The force at any point in the surface may be represented by a line tangent to the surface. This force (if the surface is curved) has a component acting toward the centre of curvature, the

magnitude of which is greater as the curvature becomes greater. This fact will help us to explain why a soap bubble which has been flattened by fanning it, or a drop of water which has become elongated in falling, tends at once to take the spherical form, that is, the form in which the curvature is the same at every point. This is also the form having the least surface for a given volume.

In Fig. 36, if the tension at *B* is resolved into two components, one toward the centre and one at right angles to it, we see that the component *BC* is greater the greater the curvature of the surface. In the case of the sphere the curvature is everywhere equal. In the tubes

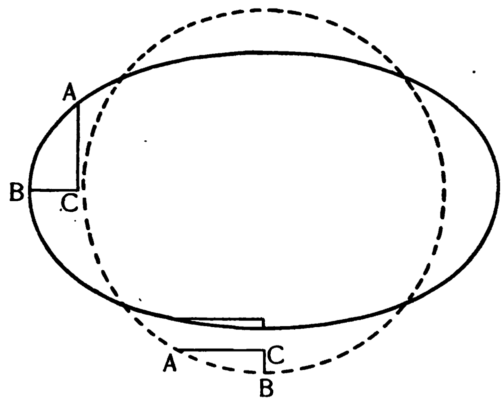


FIG. 36.

(Fig. 35) the liquid comes to rest when the difference in pressure due to a difference in height inside and outside the tube balances the tension due to the curvature.

If a drop of water be drawn into a wet tube which tapers and the tube be then held in a horizontal posi-

tion, as in Fig. 37, the curvature at *b* will be greater than at *c* (being equal to the curvature of the tube at those points), hence the water will move toward *b*. A drop of mercury in a similar tube (see Fig. 38) will move in the opposite direction. The surface at *b* (Fig. 37) must be thought of as extending along the inside of the tube to the end at *a*. When the liquid has been drawn to *a* the surface will be plane and the forces will be equal at every point in the surface. The mercury in Fig. 38 will move till the tube no longer presses it out of the spherical shape at *a*. It will not then be perfectly spherical, because of its own weight, but the curvature will be equal on its two sides and its motion will cease.

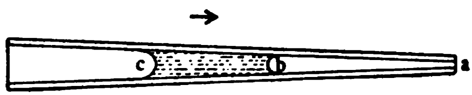


FIG. 37.

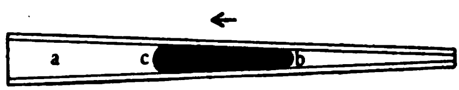


FIG. 38.

29. Floating Bodies.—If a body when immersed in water and left to itself rises toward the surface, there must be an upward force acting against gravity and that force must be greater than the weight of the body. By the second law stated on page 41 the pressure on the bottom of the body is greater than on the top, since there is a greater height of water above *b* (see Fig. 39) than above *a*.

If the body is a cube 1 cm. in height the pressure on

the top at a depth of 6 cm. is 6 grams, but the pressure on the bottom at a depth of 7 cm. is 7 grams. The resultant upward pressure is therefore 1 gram, which is the weight of the water displaced by the body. This fact was discovered by Archimedes, who gave his name to the law. The law may be stated as follows:

Archimedes' Law. — A solid immersed in a fluid is buoyed upward by a force equal to the weight of the fluid which it displaces.

Since the volume of liquid displaced by a solid is equal to the volume of the solid, and since the volume of a gram of water is one cubic centimetre, it follows that the loss of weight of a solid when immersed in a liquid is numerically equal to the volume of the solid. If the liquid weighs the same per unit volume as the solid does the solid will come to rest at any point in the liquid. If the solid is lighter, volume for volume, than the liquid the solid will rise above the surface of the liquid far enough so that the portion immersed displaces an amount of the liquid equal in weight to the weight of the solid. This principle is made use of for finding the volume of an irregular solid. The solid is first weighed in air, then weighed again immersed in water. The difference between the two weights in grams is equal to the volume in cubic centimetres of the body.

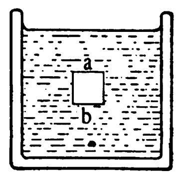


FIG. 39.

30. Liquids in Closed Vessels. — We have seen that

the pressure at any point in a liquid due to the weight of the liquid is transmitted equally in every direction. If the vessel be closed by a movable piston so that additional pressure may be applied the same law holds. Thus the pressure applied at the pumping station of a city water system is transmitted through the water in the pipes to all parts of the city, forcing the water to the tops of high buildings or at lower levels delivering it under pressure sufficient to run a water motor. The subject will be further discussed under machines.

31. Gases in Open Vessels. — Gases have some properties in common with liquids. They transmit pressure equally in all directions. Archimedes' law is true of liquids and gases alike. A feather weighing one gram is as heavy as a gram of lead, but it falls to the ground much more slowly because it displaces a large volume of air. Liquids and gases differ, however, in some important particulars. Gases have no definite surface, hence cannot have surface tension. A gas seeks to diffuse itself indefinitely because its particles are too far apart to be influenced by cohesion. Let heat be removed from the gas so that its particles are not driven apart when they chance to touch each other, or let pressure be applied to bring the particles near together, and cohesion takes effect at once, reducing the gas to a liquid.

Liquids, again, have a very definite volume. They are almost perfectly incompressible. Gases are very

easily compressed, the volume of a gas being determined by the pressure which it sustains. Every open vessel at the surface of the earth is filled with air. The weight of the air contained in any given vessel varies from day to day, as the pressure of the air varies. The weight of the air being determined by the pressure of the air above will change if there are disturbances in the air above such as to cause the air to be heaped up at certain places on the earth's surface. There are such disturbances, known as storms, which move across the country from west to east. The pressure of the air often varies as much as two or three per cent in two days during the passage of a storm. A diminution of pressure precedes a storm, a fact which is made use of to foretell the approach of storms. Let us see how the pressure of the air may be measured. Let a U tube 80 cm. high (Fig. 40) be filled about half full with mercury. The mercury will come to rest with its two surfaces on a level.

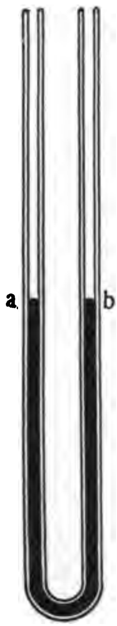


FIG. 40.

FIG. 41.

The pressure of the air on the two surfaces is equal. If now we exhaust the air from the left-hand arm, *a*, of the U tube, the pressure of the air in *b* will force the

mercury up in *a* until the weight of the air is exactly balanced by the column of mercury *ab′* (see Fig. 41), the portion of the mercury below *bb′* being equal in both arms. If the arm *a* be now closed permanently and the arm *b* be cut off to a convenient length, we have a means of measuring the pressure of the air; for as the pressure of the air increases or diminishes we have but to measure the difference in height of the two mercury surfaces. Such an instrument is called a barometer.* The air may be removed from the long arm by filling the entire tube with mercury, employing a rubber tube and funnel as shown in Fig. 42, *a*, and then inverting the tube. The same end may be accomplished by using a straight tube, *b* (Fig. 43),

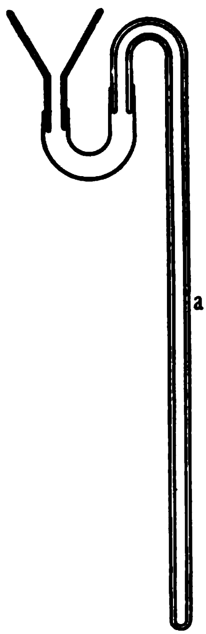


FIG. 42.

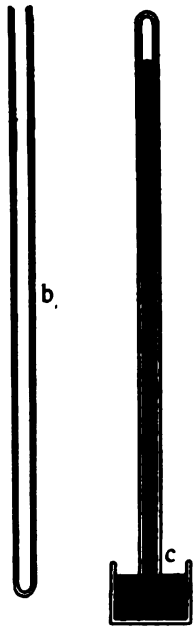


FIG. 43.

* Greek, *bar* and *metron*, weight, measure.

which is filled and then inverted in a small vessel of mercury, *c*. The mercury will fall in the tube until the pressure of the air is exactly balanced.

It will balance at about 73 cm. to 78 cm., depending on the elevation of the place and the condition of the weather.

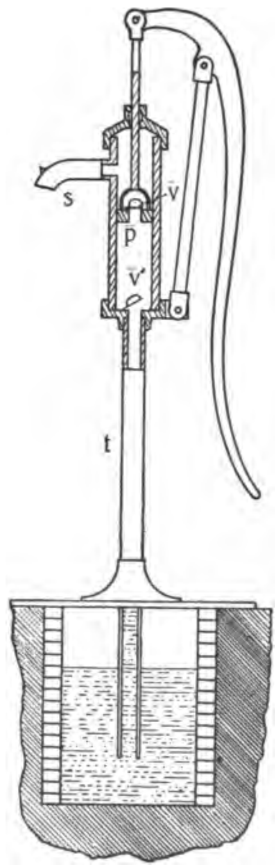


FIG. 44.

32. Pumps and Siphons. — (*a*) If water were used in a barometer instead of mercury, it would rise 13.6 times as high as mercury does, since mercury is 13.6 times as heavy as water. The common pump consists of a long tube, *t*, the lower end of which dips in water, while the upper end is provided with a close-fitting piston for removing the air. A device called a valve, *v*, allows the air to escape when the piston is pushed downward, but closes when the piston is lifted so that no air enters. The air which remains in the tube is under diminished pressure, and water will rise in the tube till the pressure of the air and water within the tube balance that of the

air on the outside. A second valve, v', is placed in the tube below the piston, so that another portion of air may be removed, and so on until the water comes above the piston, where it flows out at the spout, *s*. Such a pump will raise water about 30 feet.

(*b*) A *siphon* is used for transferring liquids from higher to lower levels over an elevation. It consists of

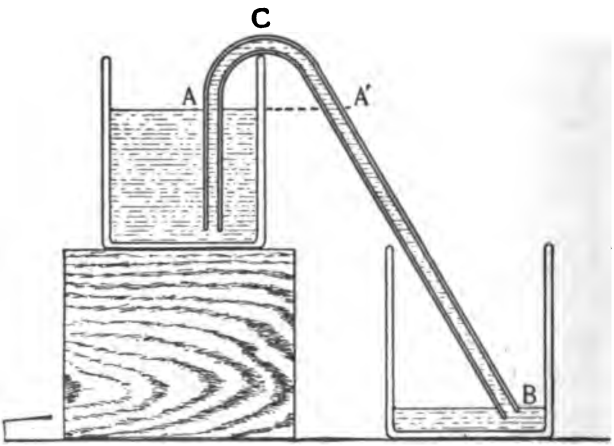


FIG. 45.

a bent tube, which is first filled with the liquid and then inverted with one arm under the surface of the liquid, the arm outside of the vessel being lower than the one within. In Fig. 45 the pressure at C in the direction of CA is one atmosphere less the weight of the column AC; the pressure at C in the direction CB is one atmosphere less the weight of the column BC. The

pressure is greater in the direction CB by the weight of $A'B$, and the liquid will flow until it is at the same level in both vessels.

33. Archimedes' Principle applies to Gases.—A solid immersed in a gas, as all bodies near the surface of the earth are immersed in air, is buoyed up by a force equal to the weight of the air displaced by it. A pound of feathers weighed out with a balance and iron weights is really heavier than a pound of lead. Placed on the two arms of the balance in air they would balance each other. In vacuum the arm carrying the lead would require some additional weight to again produce equilibrium.

34. Gases in Closed Vessels.—A gas wholly fills the vessel containing it, no matter how large the vessel may be. The pressure of a given body of gas is inversely proportional to the space it occupies, or what amounts to the same thing, its volume times its pressure is a constant quantity:

$$\textit{Volume} \times \textit{pressure} = \textit{constant}$$

$$(6) \quad vp = k$$

This statement is known as *Boyle's Law.* It does not apply to vapors, that is, gaseous substances, which by a small increase of pressure or lowering of temperature would be liquefied, for when a vapor begins to liquefy it may be subjected to pressure without much

diminishing the space occupied by the vapor. Boyle's Law is said to apply, therefore, to the so-called "perfect gases" at ordinary temperatures. Air, oxygen, and hydrogen are good examples of perfect gases.

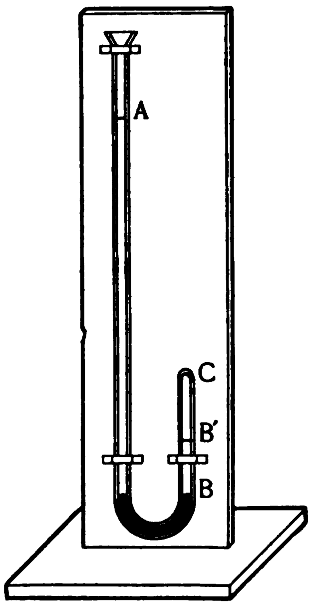


FIG. 46.

Boyle's Law may be verified experimentally by means of a bent tube, having its shorter end closed (see Fig. 46) and the bend of the tube filled with mercury. When the mercury is at the same level in both arms of the tube the pressure on the confined air is the same as on the free air, and is measured by reading the barometer. If the open arm be filled to any height, as *A*, the air in the closed arm will be compressed to a smaller volume. The height of the barometer plus the height of *A* above *B′* measures the pressure upon the confined air. If the tube is of uniform diameter the volume of the air is proportional to the length *CB*.

35. **Air Pumps.**—If part of the gas in any closed vessel is removed the remaining gas immediately ex-

pands and fills the entire space. Thus, if the piston, *p* (Fig. 47), is drawn from near the bottom of the cylinder, *C*, pushing the air before it, the air in the receiver, *R*, will expand and occupy the cylinder. If *R* is twice as large as *C*, one stroke of the piston removes one third of the air in *R*, a second stroke removes one third of the remainder, and so on, the limit of exhaustion being determined by the degree of perfection of

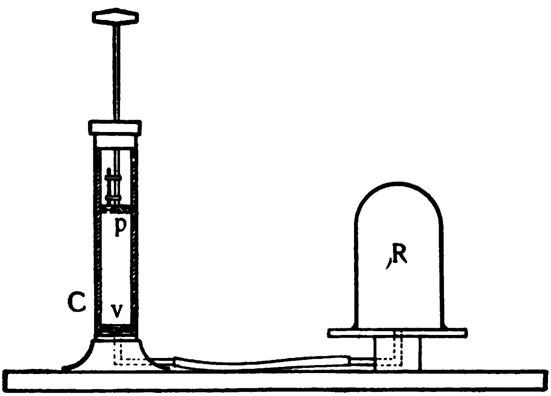


FIG. 47.

the joints about the piston and valves. Compression pumps, like the common bicycle pump, have the valves opening inward.

36. Fluids in Contact. Diffusion. — (*a*) If a vessel containing a gas be opened in a room the gas will at once expand till it is equally distributed throughout the room. This fact is evident when the gas has a

marked odor, but it is equally true in any case. It is also true that the air in the room will enter the vessel, and, in time, the contents of the vessel and the room will be identical. If, however, gases differing widely in density be placed in the same vessel, the heavier gas will tend to go to the bottom; but, unless the vessel is very large indeed, every part of the vessel will show the presence of all the gases in the vessel. The percentage of carbon dioxide present in a sleeping room is always greater near the floor than near the ceiling. This penetration by a gas of a space already occupied is called *diffusion.*

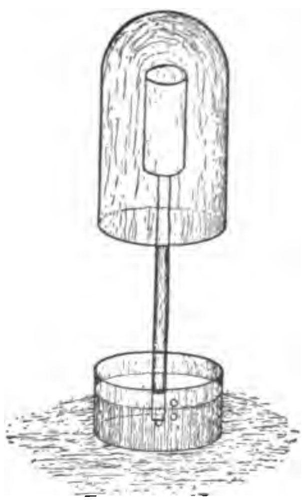


FIG. 48.

The rate of diffusion for heavy gases is less than that for light ones. If a porous battery cup (see Fig. 48) be fitted with a large cork through which passes a glass tube, the air in the cup will diffuse outward through the pores of the clay at the same rate that the air on the outside diffuses inward. Should we now dip the lower end of the tube in a glass of colored water, the water will rise in the tube to the height of the water in the glass; but if we place over the porous cup a bell glass containing a light gas like

hydrogen or illuminating gas, the hydrogen will diffuse into the porous cup faster than the air diffuses out, the pressure in the cup will be increased, and bubbles will be forced out at the bottom of the tube. If we now remove the bell glass the hydrogen will pass out through the pores of the cup faster than the air passes in, the pressure within the cup will be diminished, and the water will rise in the tube higher than in the glass and then gradually fall when equilibrium has been restored between the gases in the cup; that is, when the proportion of hydrogen and air is the same inside the cup as outside.

(*b*) When a gas is in contact with a liquid some of the gas will penetrate the liquid. Thus water which has been exposed to air contains air enough to keep fishes alive. Housekeepers know, too, that milk left open near onions absorbs the odor emitted by the onions.

(*c*) Liquids which do not separate of their own accord when mixed will mix by diffusion when placed in contact. The process is much slower than in the case of gases, but it may be observed without difficulty by introducing, by means of a thistle tube, a heavy liquid, like copper sulphate solution, to the bottom of a tall jar containing water. If the tube is withdrawn carefully, the line of separation beween the water and the heavy copper sulphate solution will be plainly marked. After the jar has stood quietly for a day or two, however, the copper sulphate will be seen to have diffused into the

water. In a week the blue color will have reached the very top of the water, thus making evident the fact which we have many other reasons for believing, namely, that the particles of fluids are in incessant motion among themselves, and are not, even in liquids, confined permanently to any particular part of the body of fluid of which they form a part.

37. **Osmose.** — If two solutions are separated by parchment paper and one wets the paper while the other does not, the former will diffuse through the paper. Solutions of crystalline substances, like sugar and salt, pass readily through such porous paper, while gums, albumen, and the like do not. This fact is often made use of to separate substances of the first class which are in solution with those of the second class. *Osmose*, as this property of diffusion of solutions through porous partitions is called, is of especial interest in the study of plant life. In winter the fluids of a tree are allowed to diminish in amount so as to prevent injury to the cells by freezing. In spring the roots take up moisture from the soil, passing it from cell to cell by the process of osmose, till the sap, often containing a noticeable amount of sugar, reaches the tip of the farthermost twig. The circulation of the sap of plants is thus carried on without that complicated system of arteries and pumps which is found in the higher animal organisms.

Exercises.

13. Why do steel journals run more easily in brass than in steel bearings?

14. Mention some cases where friction is an advantage.

15. (*a*) Do wood and other vegetable tissues shrink or swell when wet? (*b*) Is a rope longer or shorter when wet? Explain.

16. Is a three-legged or a four-legged stool the more stable on uneven ground?

17. (*a*) A steelyard has the hook which supports the weight one half inch from the fulcrum. How far apart must the one fourth pound notches be placed on the beam if the bob weighs one half pound? (*b*) Sketch a steelyard which will weigh to one ounce.

18. The force of gravity on one gram at the earth's surface is 980 dynes. What is the force on the same mass 100 kilometres above the surface of the earth?

Suggestions: (*5*) $f = \frac{Km_1m_2}{r^2}$. K is the constant of gravitation, and m_1, m_2 are constant in value, so f varies inversely with r^2, that is, $f : f' :: \frac{1}{r^2} : \frac{1}{r'^2}$ or $f : f' :: r'^2 : r^2$. $\therefore f' = \frac{fr^2}{r'^2}$.

In this example $r = 6{,}400$ *km.*
$r' = 6{,}500$ *km.*

19. Two kegs of shot, weighing 100 kilograms each, lie on the floor with their centres 40 cm. apart. What is the force tending to make them roll toward each other?

20. (*a*) The mass of the moon is $\frac{1}{81}$ that of the earth, and its diameter 0.273 that of the earth. What is the value of f (the force on one gram) at the surface of the moon? (*b*) The

mass of Mars is $\frac{10}{94}$ that of the earth, and its diameter 0.534 that of the earth. What is the value of f at its surface?

21. Why does a hydrogen balloon float in air; wood in water and not in air; iron in mercury and not in water; an iron ship in water? Can the floating of clouds in air be explained in the same way as the foregoing?

22. Why does a needle float if carefully placed in a horizontal position on water?

23. Why will the wood of a ship sunk in deep water no longer float if set free?

24. (*a*) What part of a milldam should be strongest? (*b*) Need it be stronger if the pond is a mile long than if it is but a few rods long?

25. Explain how a spoon may be filled heaping full of water.

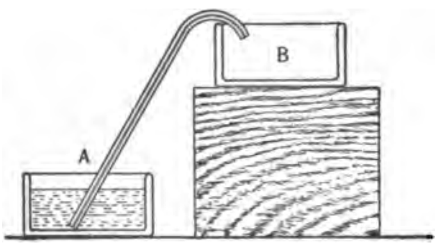


FIG. 49.

26. Why does not good letter paper make good blotting paper?

27. How do fishes rise and sink in water?

28. The capillary tube in Fig. 49 is so small that the water will rise in it to a height of 10 cm. If it were bent at a height of 8 cm., would the water from A flow over into B?

29. Why does boiled water taste "flat" even after being cooled?

30. A withered apple placed under the receiver of an air pump will, when the air has been exhausted, swell out and look plump and smooth. Explain.

31. What well-known facts are best explained on the hypothesis that matter is made up of very small parts (molecules) which are in constant motion and have in most cases some space between them?

CHAPTER III.

HEAT.

38. The Nature of Heat. — We have seen that many phenomena not easy to explain on any other hypothesis are explained by supposing that matter consists of molecules which are, so far as we know, always in motion among themselves, the amount of motion present in the molecules of any body being the cause which determines whether the body is at any instant solid, liquid, or gaseous. We have called this movement of the molecules among themselves heat, and we shall now proceed to study more in detail the phenomena connected with and the laws which govern this kind of motion.

It is to be observed that *heat*, as we shall use the term, is a motion of the particles of matter among themselves and is therefore confined to *bodies*, and may be transmitted from one part of a body to another and from one body to another body in contact with it, but cannot be transmitted through empty space. The radiations which come to us from the sun through empty space are not heat, but a form of energy, which may be transformed into heat or into some other form of energy according to circumstances. Radiant energy will therefore be treated in another place.

39. Effects of Heat. — (*a*) The most obvious effect of heat is the sensation of *warmth* produced in us when a hot body comes in contact with our skin. (*b*) A piece of iron when heat is being continually imparted to it may become too hot for us comfortably or even safely to touch, and finally it may glow, first red, then white. Most bodies which give us light are hot, light being a form of motion which in these cases results from heat and accompanies it.

(*c*) Heat changes the size of bodies. Except in special cases (when some substances pass from the liquid to the solid form, for example), the effect of heat is to cause bodies to expand. As has been remarked already, cohesion, the unknown force which draws the molecules together, is opposed by heat. If the amount of heat is increased, the molecules will be driven farther apart — the body will occupy more space.

(*d*) The change of state of bodies from solid to liquid and gaseous has already been referred to.

(*e*) Many chemical changes are conditioned upon heat. The very combustion which is the source of heat and light in a candle flame cannot occur unless the wick of the candle is first heated much hotter than the temperature of a living room. When a gust of air blows the hot gases away from the burning wick the flame is extinguished. While any adequate discussion of chemical changes is beyond the scope of this book, we may well pause here to note the difference between *physical* and *chemical changes*. The physicist finds it

convenient to consider matter made up of molecules, all the molecules of a particular substance being exactly alike and remaining identical throughout all the various physical changes to which the substance may be subjected. The chemist goes a step farther and imagines the molecule to be composed of atoms which may be unlike, though all the atoms of any one of the fundamental substances, or elements as they are called, are identical. When a molecule is separated into its component atoms, or when two or more atoms unite to form a new molecule, a chemical change has occurred.

The body to which heat is imparted usually becomes hotter, or, to use scientific terms, its temperature is raised.

40. Temperature. — The term *temperature*, while not easy to define, is used in a very definite sense. It is used to express the relative hotness (or coldness) of bodies. Any two bodies are at the same temperature if, when they are in contact, neither gains heat from the other. If one of two bodies which are in contact imparts heat to the other, the one which imparts heat is always at a higher temperature than the one receiving it. Amount of heat must not be confused with temperature. A kettle full of lukewarm water contains more heat than a cupful of boiling hot water, though the temperature of the latter is the higher. We are accustomed to form judgment of the temperature of bodies by our sensations. We call bodies hot, warm, cool,

cold, etc., without attaching to these words any very exact or definite meaning. Our sensations of heat depend so much upon the condition of our body at the time of making the observation that they cannot be relied upon to furnish exact information. Thus, if one hand be held for a few moments in ice water while the other is held in water as hot as can be borne, and then both hands be plunged at once into a vessel of blood-warm water, the latter will feel warm to the hand which has been in cold water, while it feels cold to the hand which has just come out of hot water. A common method of measuring temperature with exactness depends upon the expansion of bodies by heat.

41. Expansion. Coefficient of Expansion. — All solids expand when heated. The amount of increase in volume is usually too small to be detected without the use of some special device. A ball which passes easily through a ring when both are cold will not pass when the ball has been heated. If while the ball is hot the ring be heated also, the ball will again pass through. Railway rails are laid with space between the ends to allow for expansion in hot weather.

Liquids expand much more than solids for the same change in temperature, while gases expand still more rapidly than liquids. This would seem to be explained by the weakening of cohesion at the instant when the change of state occurs in each case. Water, from some cause not yet well understood, is peculiar in that it first

contracts on being heated (starting at the temperature of ice) and then begins to expand; after passing the *temperature* of maximum density it expands like other liquids.

The perfect gases, like air, expand at a uniform rate. The expansion of air by heating plays a most important part in the heating and ventilation of houses and in the circulation of the atmosphere. These matters will be discussed more fully later.

The expansion per unit of length of a solid for one degree change of temperature is called the coefficient of *linear expansion.* The rate of increase in volume per unit volume for one degree change of temperature is called the coefficient of *cubical expansion.* It is three times the coefficient of linear expansion. Before it can be determined we must first have a unit of temperature.

42. **Thermometry.** — Since most substances expand uniformly with increase of temperature, expansion may be used as a measure of change of temperature. Water freezes at a certain definite temperature and melts at the same temperature. Water boils at another very definite temperature (for a given atmospheric pressure). These temperatures serve admirably, therefore, as points of reference. Two scales of temperature are in common use, — the Fahrenheit, used only by English-speaking people, the Centigrade, used universally by scientists. In the former the difference of temperature between the temperature of melting ice and that of boiling water is

divided into 180 equal parts, called *degrees Fahrenheit* (180° F.), while in the latter the same difference in temperature is divided into 100 equal parts, called *degrees Centigrade* (100° C.). Fahrenheit took as his zero the temperature of a freezing mixture made of ice and sal ammoniac. It is said that he aimed thus to avoid negative readings, as this was the lowest temperature observed by him at Dantzig in the year 1709, when he was making his experiments. The zero Fahrenheit is 32° below the freezing point of water, which makes the boiling point of water $180° + 32° = 212°$ F. Celsius, who devised the Centigrade scale, used the freezing point of water as his zero, thus making the boiling point of water 100° C. The change covered by a Fahrenheit degree is therefore $\frac{100}{180} = \frac{5}{9}$ that covered by a Centigrade degree. To reduce any number of Fahrenheit degrees to Centigrade degrees we must therefore multiply by $\frac{5}{9}$. The *reading* on Fahrenheit scale is 32° at the freezing point; hence to reduce Fahrenheit reading to Centigrade subtract 32° and multiply by $\frac{5}{9}$.

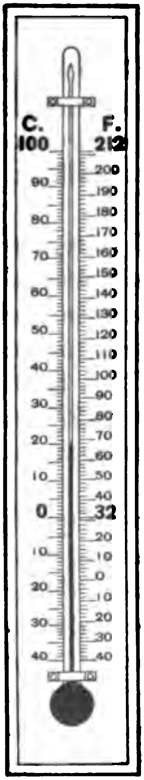


FIG. 50.

(7) Reading C. $= \frac{5}{9}$ (Reading F. — 32°).

(8) Reading F. $= \frac{9}{5}$ Reading C. + 32°.

Fig. 50 makes the matter plain.

The thermometers most used consist of a glass tube of very small bore, terminating in a bulb which contains mercury or alcohol. The tube was sealed at a temperature at which the liquid filled both bulb and tube. At the lowest temperature for which the thermometer is designed the liquid still extends a short distance into the tube. A comparatively small expansion of the liquid in the bulb causes a movement of the fine thread in the tube of sufficient amount to be easily seen. The method of marking a thermometer, or calibrating it as it is called, is explained in Part II.

The glass which contains the mercury expands only about one seventh as much as the mercury. Alcohol expands more than five times as much as mercury, and therefore makes a more sensitive thermometer than does mercury. It does not freeze even at the lowest arctic temperatures, while mercury solidifies at — 39° C. Alcohol is not opaque, however, even when colored, so that an alcohol thermometer is more difficult to read than a mercurial thermometer.

Air expands four times as much as alcohol, and is, therefore, very suitable for measuring small changes of temperature over a small range. The air thermometer has the disadvantage that its readings vary with the atmospheric pressure, so that a reading of the barometer must be made whenever the temperature is observed.

The difference in expansion of two metals is sometimes made use of in constructing thermometers. If a strip of iron and a strip of zinc are riveted together

at several points so that the compound bar is straight when cold, the two metals will expand when heated and the bar will be curved as shown in Fig. 51. If the end at *a* is rigidly fastened to a support, the free end may be made to transmit its movements to a pointer, which will indicate the temperature upon a dial. The dial may be calibrated by comparison with a good mercurial thermometer.

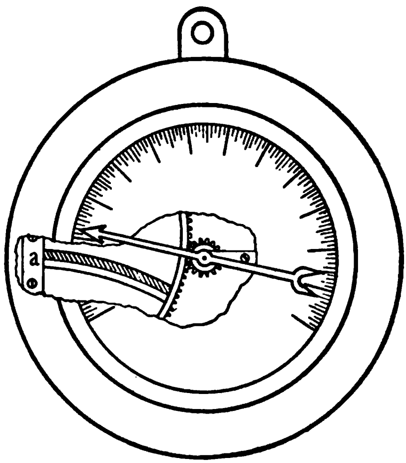


FIG. 51.

The same sort of compound bar may be made to close alternately two electric circuits when the temperature of the room varies any given amount above or below the normal, thus turning off or on the steam, and so automatically regulating the temperature of the room. Such a device is called a *thermostat.* It is sometimes used to give an alarm of fire if the temperature reaches a dangerous point.

43. Law of Charles. Absolute Zero. — It was proved by Charles in 1787 that *at constant pressure the volume of a gas increases uniformly with its temperature.*

The rate of increase is the same for all gases and at 0° C. this rate is $\frac{1}{273}$, or 0.00366.

If the volume is kept constant the pressure increases at this same rate. A quantity of air which, with the barometer at 760 mm., occupies 273 c.c. at 0° C. would, if the pressure were kept constant, occupy 373 c.c. at 100° C. and 173 c.c. at —100°. If, then, we mark an air thermometer in degrees of the same length as for the Centigrade scale, but call the freezing point of water 273°, we shall have a scale on which the readings are directly proportional to the volume of the gas. Such a scale is called an *absolute scale.* Its zero is at —273° C., a temperature which of course can never be reached in the laboratory. It is believed that the spaces between the stars are at the absolute zero, however, since there is no evidence for the presence of matter in any appreciable amount in the space between us and the sun, except within a few hundred miles of the earth and a few thousand miles of the sun.

To express Centigrade readings in absolute we have only to add 273 to the Centigrade reading. Thus 50° C. = 323° absolute.

44. Quantity of Heat.—The quantity of heat required to raise the temperature of a given body one degree differs with the kind and quantity of matter comprising the body. The unit of heat is the heat required to raise the temperature of one gram of water 1° C. The unit of heat thus defined is called the

calorie. It requires 200 calories to raise the temperature of 20 grams of water 10°.

45. Heat Capacity. Specific Heat. — The quantity of heat (measured in calories) required to raise the temperature of any body 1° C. is the *heat capacity* of that body. The quantity of heat required to raise the temperature of one gram of any substance 1° C. is called the *specific heat* of that substance. It is a quantity which is very different for different substances, but is constant for any given substance through a moderate range of temperature. The specific heat of iron, for example, is 0.113. The heat capacity of an iron kilogram weight would be $1{,}000 \times 0.113 = 113$ calories, while the heat capacity of one litre of water (weight one kilogram) is 1,000 calories, or nine times as much. If one kilogram of iron at 100° C. were plunged into one kilogram of water at 0° C., the temperature of the water would rise 10° C. and that of the iron would fall 90° C., since the two substances in contact tend to come to equilibrium. It follows that by putting together known masses of two substances at different (known) temperatures, we may determine the relative specific heats of the substances; and, if the specific heat of one is known, the specific heat of the other may be found at once. For the quantity of heat transferred is $H = mst$, where m is the mass, s the specific heat, and t the change of temperature. But the heat lost by one body exactly equals the heat gained by the other.* That is:

* Proper precautions must be taken to prevent loss to the air or other bodies.

$H = H'$, whence $mst = m's't'$ and

$$(9) \quad s' = \frac{mst}{m't'}$$

This method of determining specific heat is called the method of mixtures.

46. Fusion. — If we heat a solid like ice or wax it will rise in temperature for a time and then begin to melt, the temperature remaining constant after melting has once begun until the whole is melted, when the temperature again begins to rise. If the liquid is now allowed to cool slowly it will begin to solidify at the same temperature at which it melted. There are two important differences to be noted in the solidification of water and wax. The water solidifies in crystals, that is, prism-like bodies of definite geometrical form, while the wax exhibits no structure whatever.

The water, probably because the arrangement of the molecules in a particular geometrical form leaves more space between them, expands in solidifying, while the wax contracts. Those metals, like iron and brass, which expand on solidifying make excellent castings because they fill out the mold. Gold or silver, on the other hand, must be minted or stamped to give clear, sharp outlines.

The fact that water expands on freezing, coupled with the fact that it contracts when warmed from 0° to 4° C., has important bearings on the life of all marine animals in particular, and, in less degree, upon the life

of all animals outside the Torrid Zone. If ice were heavier than water the first ice formed on a lake in autumn would sink to the bottom, another layer would form and sink, and the process would continue till the lake became a mass of ice. In spring the top layer would melt, but since water transmits heat downward very slowly the lower portions would probably never thaw out. Let us consider what actually happens. The surface water in autumn cools down to 4° C., its temperature of maximum density, and sinks to the bottom. Freezing is thus delayed till the entire body of water is at 4°, when the portion at the top cools to 0° and freezes, and the water next to the layer of ice is slowly cooled to the freezing point and added to the first layer formed, the entire layer of ice seldom extending more than a few feet at most downward, while the great body of water never freezes.

47. Vaporization. — Most liquids, if left exposed to the air, gradually disappear. The molecules within the liquid, while they are all in motion, have not all the same velocity. Some of those which are moving fastest make their way through the surface and go off as vapor; the liquid slowly evaporates. Any increase in temperature will increase the number of particles which have velocity enough to penetrate the surface film and so increase the rate of evaporation. It is obvious, too, that a large exposed surface is favorable to rapid evaporation. If the air above the surface is at rest it will

soon become *saturated;* that is, it will be so full of vapor that the gaseous particles constantly striking the surface of the liquid will make their way into the liquid as fast as those within escape. When the space above a liquid is saturated, evaporation and condensation are going on at the same rate, which amounts to the same thing as if evaporation ceased. Evaporation is hindered by the pressure of the air on the surface of the liquid; it therefore goes on very rapidly in a partial vacuum, a fact which is sometimes made use of in evaporating the water from sugar sap. The pans are covered with a tight-fitting hood, which terminates in a pipe leading to an air pump. This pump removes the vapor at the same time it diminishes the pressure, thus producing very rapid evaporation.

Evaporation goes on from the surface of solids also, but in less degree than from liquids, since the molecules have less freedom of motion. A block of ice left on the shady side of a building on the coldest day in winter will slowly evaporate, as may be seen from the fact that its corners become rounded. A lump of camphor left exposed to the air will slowly evaporate, though it does not melt. The evaporation of a solid is called *sublimation.*

48. Ebullition. — Evaporation occurs only from exposed surfaces. If a liquid be heated hotter and hotter, a point will in general be reached where vapor bubbles will form on the surface of the vessel where

heat is applied and in the body of the liquid itself. These bubbles will at first form, rise into the cooler portions of the liquid and collapse, giving forth at times a note like the "singing" of a teakettle. When the whole of the liquid reaches the temperature at which the bubbles formed they will rise rapidly to the surface. The liquid is boiling, and if heat is still applied it will continue to boil without any rise in temperature till all the liquid has been vaporized. The boiling point of a liquid is definite for a given pressure of the atmosphere, but falls with diminished pressure. On the top of Pike's Peak water boils at 187° F., while at sea level it boils at 212° F. It is not worth while, therefore, to attempt to cook vegetables by boiling in an open vessel at such high altitudes.

In a locomotive boiler carrying a steam pressure of two hundred pounds to the square inch water boils at 382° F. This fact accounts for the terrible severity of scalds produced by escaping steam following the bursting of a boiler in a railway collision.

49. Distillation. — The fact that different substances boil at different temperatures makes it possible to separate two liquids by heating the mixture to a temperature a little above the boiling point of the more volatile liquid, which will then go off in vapor. The vapor is conveyed in pipes through cold water, where it condenses. Alcohol and water are separated in this way. Since evaporation occurs at temperatures below the

boiling point, a small portion of water vapor goes over with the alcohol. The process may be repeated, however, and the portion first collected from the second distillation will be almost perfectly pure alcohol. A still is shown in Fig. 52.

50. Evaporation a Cooling Process. Latent Heat. — We have seen that when a liquid is evaporating it is the molecules having the highest velocities which leave

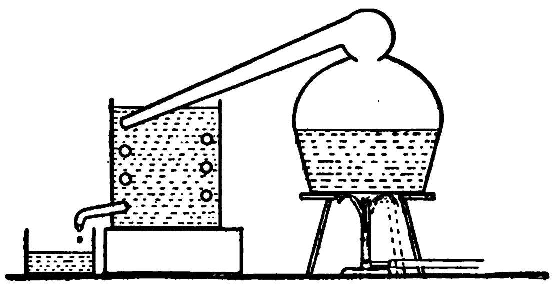


FIG. 52.

the liquid. Since the temperature of the liquid depends upon the average velocity of its molecules, it is evident that the liquid must be cooled by evaporation. If we wet the bulb of a thermometer with water the temperature of the bulb will fall to a point several degrees below the temperature of the room, the amount it falls depending in part upon the amount of moisture in the air. Indeed, the difference in reading between the wet and dry thermometers at any given temperature is an index to the humidity of the air. When

water is boiling the heat supplied to keep it boiling makes good the loss of heat by vaporization. Since this heat produces no sensible rise in temperature, it has been called *latent* (hidden) *heat.* When the vapor liquefies, the heat which kept the molecules apart against cohesion becomes again sensible heat. The amount of heat required to vaporize one gram of water at 100° C. is 536 calories, or 5.36 times as much as would heat the same quantity of water from 0° to 100°. This quantity, 536 calories, is called the latent heat of vaporization of water at 100°. It is somewhat more at lower temperatures and less at higher.

A similar loss of sensible heat occurs in the change of state of water from solid to liquid, though much less in amount. The latent heat of fusion of water at 0° C. is 80 calories. This is equivalent to saying that the amount of heat required to melt a quantity of ice at 0° would raise the temperature of the same quantity of water from 0° to 80°.

51. Conduction. — If one end of an iron bar is heated in the fire the other end will, after a time, grow hot also. The motion imparted to the particles by the hot coals is soon passed along to adjoining particles, which in turn pass it to those farther away. This process is called *conduction.* Substances differ greatly in their power of conduction, the metals being good conductors, while wood, wool, cotton, water, and air are poor conductors.

52. Convection. — Fluids, like air and water, while they are poor conductors, possess two properties which make it possible for them to convey heat with considerable rapidity: (1) they have freedom of motion among their particles, and (2) they expand a great deal when heated. The result is that if a fluid comes in contact with a heated surface which is below it, it begins to expand as soon as heated, and, being lighter than the surrounding fluid, is forced upward by the latter, which takes its place, thus keeping up a constant circulation by means of which heat is *conveyed* to great distances. The process is therefore called *convection.* The pointed shape of a candle flame is due to the convection currents formed in the air (see Fig. 53).

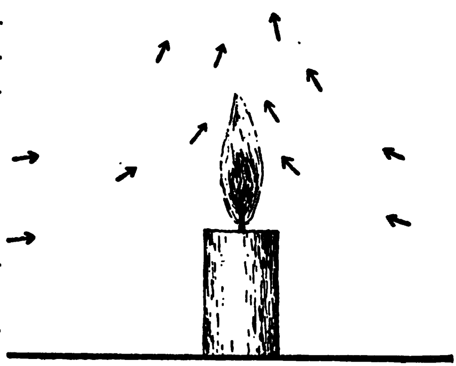


FIG. 53.

53. Heat and Human Life. — The temperature of the human body does not vary in health more than a degree or two from 98.6° F. Yet we live in a climate where variations of 50° occur within twenty-four hours and the extreme variation in a season may reach 150°. It may be profitable then, by way of review, to see how

the laws of heat which we have been studying bear upon some of the problems which face us every day.

54. **Weather.** — Wind, temperature, rain, clouds, change from day to day and furnish a never-failing topic of conversation. Everybody is interested in the weather, yet most of us do not observe weather changes as carefully or understand them as fully as we might. Winds may be simple convection currents flowing toward the hottest region of the earth. Such are the "trade winds" which flow toward the equatorial regions and the sea breezes which blow toward the land in day time, alternating with land breezes toward the sea at night.

The winds which most affect the weather in temperate regions, particularly away from the sea, are the cyclonic winds which blow toward an area of low barometric pressure. Such low areas move across the country from west to east, followed by regions of high pressure, about once in five or six days on the average; and it is by following the movement of such storm areas that the officials of the Weather Bureau are able to predict in advance the weather changes which are likely to occur within the next thirty-six hours. To the east of a low area the winds blow from the east and south and are warm and laden with moisture. As the low area passes we encounter west and northwest winds and the temperature falls. The cooler north winds condense the moisture which the south winds

bring, hence we have the most clouds and rain when the barometer is low, followed by fair and cooler weather as the barometer rises. Absorption of solar radiation by day and loss by radiation at night will be better understood when we have studied radiant energy.

We protect ourselves against weather changes in two ways: we wear clothing and live in houses.

55. Clothing. — The heat of our bodies is maintained by chemical changes which occur in the bodily tissues. These changes are in the nature of a disintegration or destruction of the tissues, the loss being constantly made good from the food which we eat. The supply of heat thus furnished is very irregular, but the body is provided with an automatic regulating device in the pores of the skin. Moisture is constantly exuding from these pores and evaporating, thus cooling the body. When the body is warm the pores open and we perspire freely; when we are cold the pores close. Clothing checks the circulation of air near the skin, and, being made of non-conducting materials, retards the escape of heat from the body.

56. Heating and Ventilation. — When a house is heated with a hot-air furnace taking the air from out of doors the rooms do not lack for fresh air, but the heating of the house is not likely to be as satisfactory as when the cold air is supplied to the furnace from the coldest parts of the house, such as a hallway or a

northwest room. With this arrangement the cold air flows down to the furnace while the hot air from the registers finds its way to these colder parts of the house to supply the place of the cold air. Fresh air may be admitted to any part of the house through windows or

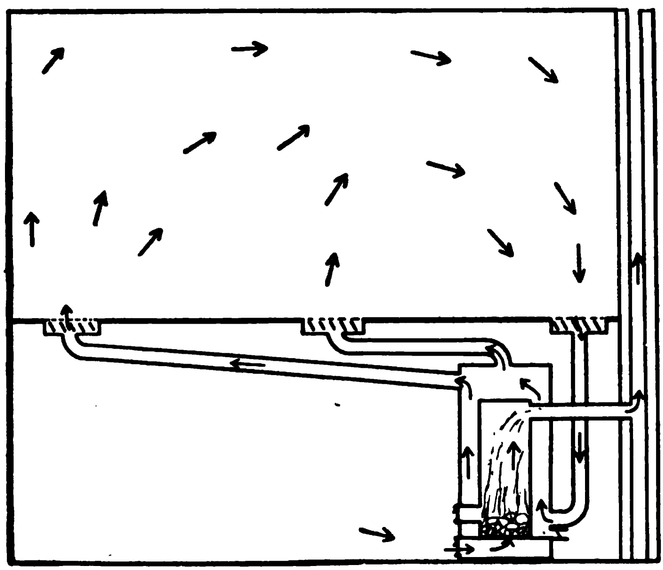


FIG. 54.

through openings provided for the purpose. The circulation is shown in Fig. 54.

When a large room is to be heated by a stove the end may be often much more satisfactorily accomplished if, instead of leaving the circulation of air in the room to chance, with the chances in favor of stagnation and a

wide variation of temperature, the stove be surrounded by a large drum, open at top and bottom and reaching from near the floor to a point some distance above the top of the stove. The drum may be in two parts hinged together for convenience in getting at the stove. A little reflection will show that the drum will shield that portion of the room nearest to the stove from excessive heat, while, by setting up convection currents, it brings the cold air from the farthest parts of the room to the stove, where it is heated and returned along the ceiling, thus heating the room uniformly and much improving the ventilation at the same time. In buildings heated by steam or hot water the hot fluid circulates through the pipes by convection, and the heat is distributed by placing in each room a radiator consisting of a coil of pipe having an amount of surface suited to the size of the room. In the matter of ventilation these systems are inferior to the hot-air furnace. Pipes are sometimes so placed in the ventilating flues, however, as to create a draught and ventilate the room.

A combination of both systems is now being used in cities where power is easily procured in which air is driven over hot steam pipes (in summer over ice) by means of a fan run by an electric motor or other power and forced through registers to all parts of the room. In this way a large church may be thoroughly ventilated and heated in half an hour during the coldest weather, or made comfortably cool with a few hundred pounds of ice in summer.

Exercises.

32. It is desired to obtain the temperature of a liquid. What error might arise (*a*) if the thermometer were read soon after inserting in the liquid; (*b*) if the thermometer were inserted only far enough for the liquid to cover the bulb; (*c*) if you took the thermometer out of the liquid to read it? Explain.

33. Why is there sometimes frost on the grass in autumn when a self-registering minimum thermometer records no lower than 36° F.?

34. Explain how the height of a mountain above sea level might be determined with a thermometer. Would it be necessary to make observations on more than one day to obtain accurate results?

35. What is normal blood temperature (*a*) in Centigrade degrees; (*b*) in absolute degrees Centigrade; (*c*) in absolute degrees Fahrenheit?

36. Why is extreme heat harder to bear in moist climates than the same temperature in dry climates?

37. Why is extreme cold felt more in moist climates than in dry?

38. Why do boiler makers use red-hot rivets for fastening the plates of a boiler together?

39. Why do freezing water pipes burst?

40. A certain metal, if thrown cold into a vessel of the same metal molten, floats. Will this metal make sharp castings?

41. Why do we say that when smoke falls from the chimneys it is likely to rain?

42. (*a*) Why does a pitcher containing cold water "sweat" in summer? (*b*) A kerosene lamp set away clean is often found covered with a film of kerosene. Is this fact analogous to the "sweating" of the pitcher of water?

43. An air thermometer (see Fig. 55) was filled with water to the point *A*. When dipped in a beaker of hot water it first fell to *B* and then rose to *C*; taken out and plunged in cold water it rose to *D* and then fell. Explain.

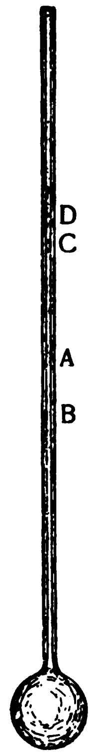


FIG. 55.

44. In hot weather metal objects feel hotter than wooden objects which are beside them in the sun. Are they really very much hotter, and if not, why do they seem so? What is the case in cold weather?

45. Ice houses are made with double walls having sawdust between them. Would a wall of logs the same thickness as the double filled wall serve as well?

46. How would it be possible at high altitudes to cook vegetables by boiling?

47. Tubs of water are often placed in a cellar to prevent vegetables from freezing. Explain.

48. Why are the extremes of heat and cold greater in the middle of a continent than on the coast?

49. Why do clothes dry faster on a windy day than when the air is calm?

50. A litre of air at 746 mm. pressure and 20° C. has what volume under standard conditions; namely, 760 mm. and 0° C.?

51. An iron rail is 30 feet long at 20° F. What is its length at 120° F.? Iron expands 0.0000064 of its length for each degree.

CHAPTER IV.

ELECTRICITY AND MAGNETISM.

57. Introductory Remarks. — While the laws governing the group of related phenomena which we designate as electrical and magnetic are very well known, it must be borne in mind that we are as yet not able to give any very satisfactory answer to the question: What is electricity? To be sure, we have not the slightest notion what cohesion is or what gravitation is. The words cohesion and gravitation are but symbols for what we choose to designate the causes of two classes of phenomena. Our inability to define more exactly these forces need not stand in the way of our study of the phenomena connected with them. Indeed, it is only by frankly admitting our ignorance and seeking to find out as large a body of facts as possible that we may ever hope to reach a knowledge of the ultimate nature of such fundamental entities as matter, electricity, and gravitation.

The average student has not so large a fund of observation and experience from which to illustrate electrical phenomena as he had in the subjects treated in preceding chapters. He will find it necessary, therefore, to observe closely and remember carefully every new fact which comes in his way.

There is every reason why all intelligent people should wish to understand that wonderful agent which is to play so important a part in the life of the century now opening, if indeed, as we must suppose, the discoveries of the last fifty years have but opened the way to still more wonderful discoveries yet to be made.

58. **Magnets.** — It was long ago discovered that certain specimens of iron ore had the power to attract bits of iron to themselves. From the country where these "lodestones" * were most frequently found — Magnesia — they were called *magnets.* It was very early known, too, that if a bar of steel be rubbed with a lodestone or natural magnet it will itself become a magnet, and that such artificial magnets will, when suspended, point nearly north and south. An elongated mass of lodestone will do the same thing, but the fact is much more readily observed with a long and slender bar like a compass needle. The mariner's compass is such a bar or needle supported on a pivot so as to move freely in a horizontal plane. It was in use before the time of Columbus.

59. **Magnetic Poles.** — If we dip a slender bar magnet, like a magnetized knitting needle, into iron filings, the filings will adhere to the magnet only near the ends, *AD* and *CB* (Fig. 56). If we cut off that portion at each end to which the filings adhered and dip the magnet again in filings, the filings will again adhere

* *Lode*, a vein of iron.

to the ends of the portion DC which remains. It would appear, therefore, that while the magnetism manifests itself only at the ends of the bar, it is really present in other parts of the bar, or it would have disappeared when the ends were cut off. Had we cut

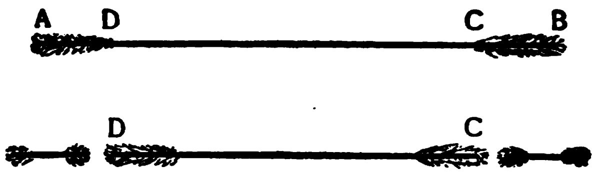


FIG. 56.

the bar at its middle point we should have obtained two magnets, each attracting filings at its ends. Moreover, the two pieces, AD, CB, cut from AB behave exactly like the original magnet. The ends of a bar magnet are called *poles*. If we suspend a bar magnet, and, after allowing it to come to rest, mark the end which points to the north, we shall find that end which we have marked will always be the north-seeking end

FIG. 57.

of the magnet; that is to say, the two poles of the magnet are unlike. If A was a north-seeking pole in the magnet shown in Fig. 56, the arrangement of poles in the fragments would be as shown in Fig. 57.

60. Attraction and Repulsion.—When the N-end of a magnet is brought near the N-end of a suspended

magnet, like a compass needle, the N-end of the needle is repelled and the S-end attracted. In like manner, a S-pole repels a S-pole and attracts a N-pole. Briefly stated, like poles repel, unlike attract.

61. Law of Force. — We may define unit pole as follows: let two poles be of equal strength, such that at a distance apart of one cm. they repel each other with a force of one dyne, they are unit poles. Coulomb proved that the law of force is similar to the law of gravitation: two magnetic poles of strength m units and m' units respectively at a distance r cm. apart repel or attract each other with a force proportional to the product of their strengths divided by the square of the distance apart.

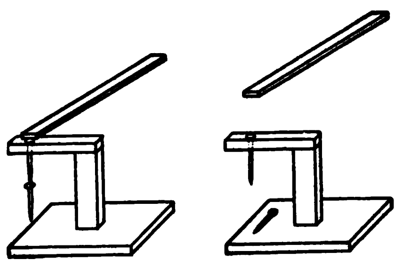


FIG. 58.

$$(10)\ f = \frac{mm'}{r^2}$$

62. Induced Magnetism. — Magnets are commonly made by rubbing steel bars upon a magnet. A piece of soft iron needs only to be brought near a magnet to become itself a magnet for the time being. Thus the nail in Fig. 58 attracts the second nail and holds it so long as the former is in contact with the magnet. If we remove the magnet the nails no longer cling together. Hard steel is not so easily magnetized as

soft iron, but the steel retains its magnetism after removal from the magnet, while the iron does not. This fact is analogous to the fact that it is less easy to put an edge or a point on hard steel than on soft iron, but the edge or point will be retained correspondingly better by the steel. Magnetic attraction occurs only between magnets in every case, for the pieces of soft

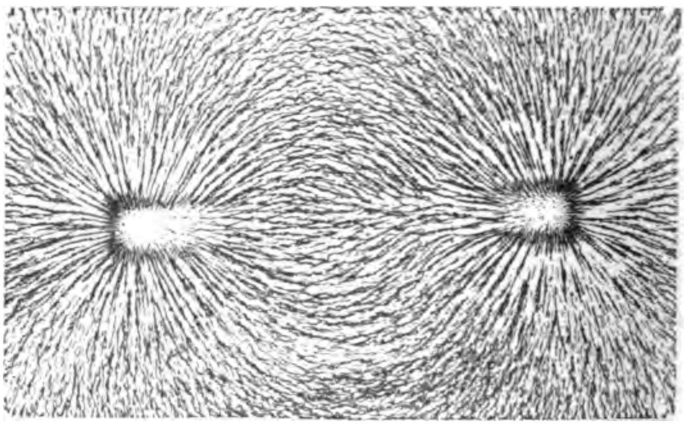


FIG. 59.

iron all become magnets by *induction*, as it is called, and are then attracted. The induced pole nearest the magnet is always of opposite kind to that which induces it. It is evident, therefore, that a magnet cannot repel soft iron, since unlike poles always attract.

63. The Magnetic Field. — A magnet moves a magnetic needle without touching it. The needle may be

enclosed in a glass bottle. The magnet still acts upon it. The air may be exhausted from the bottle. The magnet acts exactly as before. This space surrounding a magnet, within which a magnet cannot come without being influenced to take a particular direction and within which also every bit of soft iron becomes temporarily a magnet, is called the *magnetic field.* It extends

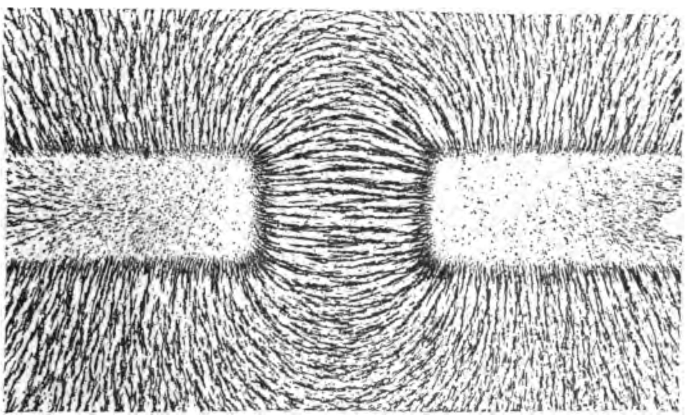


FIG. 60.

in all directions from the magnet, growing weaker as we go farther from the magnet, in accordance with the law of force stated in Section 61.

If we place a piece of glass or stiff cardboard on a magnet, dust iron filings over the glass and gently tap it, the iron filings will arrange themselves end to end, the direction of each bit of iron being the direction of the resultant of the two forces from the two poles of

the magnet. Fig. 59 is taken from a photograph of such a chart of the magnetic field made by using a photographic plate instead of the piece of glass.* Fig. 60 shows the field between unlike poles, while Fig. 61 shows the field between like poles.

64. Magnetic Induction Explained. — We have seen that a magnetized knitting needle may be divided

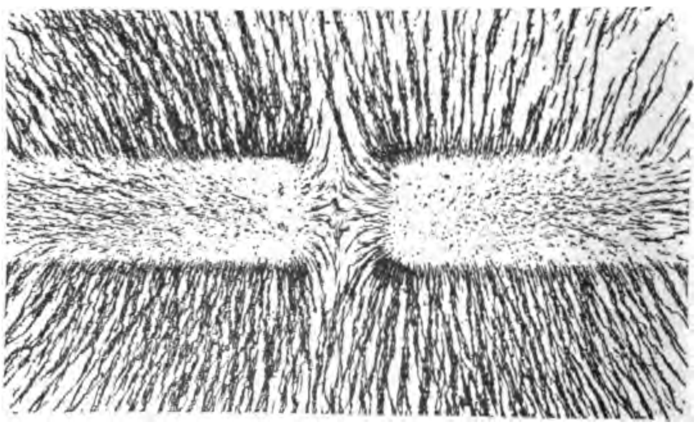


FIG. 61.

into a great many short parts, and each little piece will be a magnet. This suggests that every molecule of a magnet is itself a magnet — nay more, that every molecule of iron is a magnet. In a bar of soft iron the molecules, being magnets and more free to turn than in the

* The work was of course performed in a dark room. When the filings were in position an electric light was turned on for a second. The filings were then brushed off, and the plate developed.

steel, take the most stable position possible; that is, every N-pole will get as near a S-pole as possible. In this position the force of any N-pole is neutralized by the S-pole near it as far as producing any effect outside the iron is concerned. If a strong magnet is brought near the iron it turns the little molecular magnets all one way, and the molecules at the ends will attract, while those at intermediate points in the bar still neutralize each other. Fig. 62, *a*, shows the arrangement

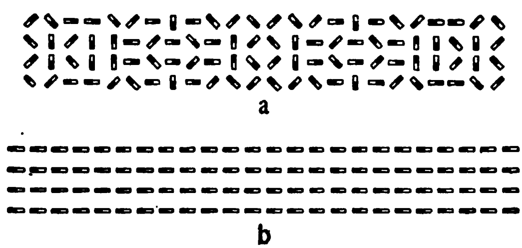


FIG. 62.

of the molecules before the magnet is brought near; *b*, while the magnet is in the neighborhood.

When a steel knitting needle has been given a single stroke with a magnet it is a weak magnet, since only part of the molecules have been arranged. By further stroking all may be made to point in the same direction, after which no magnet, however strong, could induce any more magnetism in the needle than it already possesses.

65. Magnetic Substances. — We have hitherto spoken only of iron as a substance capable of magnet-

ization. It is not, however, the only magnetic substance. Nickel and cobalt are magnetic to a small degree as compared to iron, but to a large degree as compared to air, water, and most other substances. Most substances are so slightly magnetic that their magnetic properties can be detected only by means of very delicate instruments.

66. The Nature of the Magnetic Field. — While air or other substances which may happen to lie within the field of force of a magnet are influenced by that field, they are in no way necessary to its existence. The force which is transmitted from a magnet to a compass needle is not conveyed by means of the air. We cannot think of a body being set in motion without contact, direct or indirect, with the body which sets it in motion. It is believed that all motions which are transmitted from one body to another not in contact with it are conveyed through a medium which pervades all space, even the space occupied by matter. This medium which is very elastic is called the *ether*. The ether in a magnetic field is supposed to be in a state of strain due to a stress, which the magnet, in some way, puts upon it. We are familiar with the transference of stress in an elastic fluid from one body to another in the fluid. The bullet in an air gun has motion imparted to it in this manner. The stress in a field between two unlike poles would drive a N-pole along the lines of force indicated in Fig. 60 away from the N-pole toward the

S-pole. A N-pole placed between the two N-poles in Fig. 61 halfway between them would be in unstable equilibrium, but once started toward either magnet would move along the lines of stress to the S-pole of that magnet toward which it started.

The action between two magnetic poles could be explained by supposing the lines of force to be stretched elastic cords which are all the time trying to contract, and which constantly repel each other. A strong field of force would be one in which these lines lie very close together. In a field of unit strength there is assumed to be one line passing through every square centimetre of a plane drawn at right angles to the lines. It will be seen that the lines may be thought of in two ways, either as mere geometrical lines indicating by their direction the direction of the force at every point, or as physical cords which tend to shorten and widen as a stretched rubber cord does. The direction of the lines is always supposed to be from a north to a south pole. Every line is a closed curve passing in at one end of the magnet and out at the other.

67. Effect of Heat on a Magnet. — A steel needle which has been magnetized will lose its magnetism if heated red hot. This is easily explained if we remember that the molecules tend to arrange themselves promiscuously. The agitation due to heat sets them free, they return to their natural positions, and the needle is demagnetized. Jarring a magnet has a similar effect.

68. Magnetism of the Earth. — The earth acts like a great magnet, having its poles at some distance from the geographical poles, the one in the northern hemisphere being in the northern part of Hudson's Bay.

The magnetic needle points due north at places on a line which coincides roughly with the meridian which passes through that pole. At places in the Eastern States the needle points west of north. At places west of Central Ohio the needle points east of north. This declination must be known for any place before the compass can be used to tell directions accurately. In cities the compass is now of little use in surveying because of the presence of large masses of iron, such as gas and water pipes.

The horizontal declination of the needle, that is, its variation from true north in different parts of the world, was known to Columbus, who recorded the declination for a large number of localities in the Atlantic Ocean.

Compass needles are usually balanced so as to swing only in a horizontal plane. If a needle is allowed to point in the direction of the lines of force of the earth's field, it will dip downward at the north end in the northern hemisphere.

69. Electric Charges. Attractions and Repulsions. — It was known to the ancient Greeks that amber, a fossil gum, when rubbed, would attract light

bodies like bits of wood or pith. The Greek word for amber — *elektron* — has given us our words electrify, electricity, and so forth. A body which has by contact with another body of different material acquired the power to attract light bodies is said to be electrified. If a glass rod be rubbed with silk the rod will attract bits of pith, paper, and other light materials. The particles of pith cling to the rod for a time, then jump away as if repelled. A light ball of pith or cork (see Fig. 63) hung by a silk thread from an insulating support will be attracted to the rod, and after contact will be repelled. The ball will be attracted to a stick and will attract small particles of pith. The ball has become electrified by touching the electrified rod.

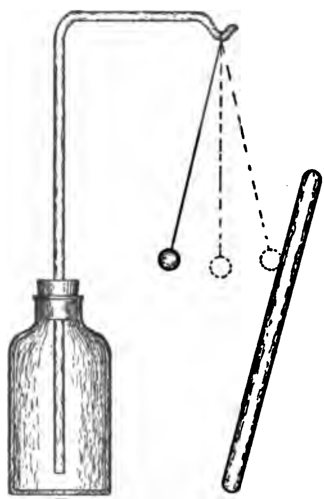


FIG. 63.

When the rod was rubbed with the silk, the silk was electrified as well as the rod. The ball which has been charged by touching the glass rod will be attracted by the silk but repelled by the rod. We say, therefore, that there are two kinds of electrification, and we call the kind on the glass positive or plus, that on the silk negative or minus. When a vulcanite rod or a bar of

sealing wax is rubbed with flannel or fur, the rubber or wax is negatively electrified, the flannel or fur positively. Bodies having like electrification repel, those having unlike attract. Fig. 64 shows diagrammatically the action between pairs of rods, one of which is electrified and suspended in a sling of paper so that it may swing, while the other is electrified and held near it. The black rods are vulcanite, the light ones glass.

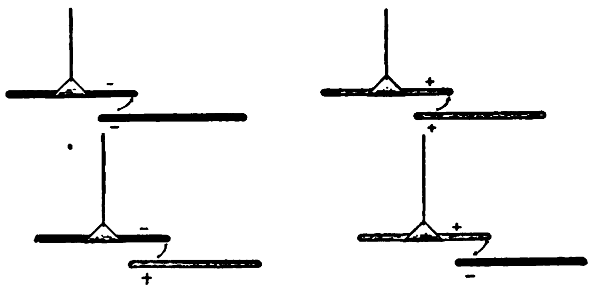


FIG. 64.

70. Conductors. Insulators.—Dr. Gilbert, physician to Queen Elizabeth, examined a large number of substances and found that under the same conditions certain substances could be electrified, others apparently could not. He called them "electrics" or "non-electrics," according to his results. Most of his work has proved correct; but, like all men of science, he taught us also by his mistakes. He tried to electrify a metal rod while holding it in his hand, and failing, he concluded metals were "non-electrics." If he had held the metal rod in a pad of silk he would have obtained an oppo-

site result. His facts were correct, for the body carried the charge to earth from the metal rod, but his theory was wrong because he did not reckon in the effect of his hand on the metal rod. We now classify bodies as conductors and non-conductors or insulators. A charge on one part of any body which is an insulator does not spread to other parts of the body. A charge on a conductor spreads instantly to all parts of the conductor. The metals are good conductors, as also is water-vapor, wet cotton, wet wood, moist air, or any wet substance. On the other hand, dry cotton, dry wood, and dry air are good insulators, that is, poor conductors. The earth is a good conductor, so then any charged body which is connected to the earth by another conductor, such as a wire, a gas pipe, or the human body, will at once lose its charge. We find it necessary, therefore, when we wish to charge a conductor, to *insulate* it from the earth by placing it on a support of glass, rubber, paraffine, varnished wood, or the like.

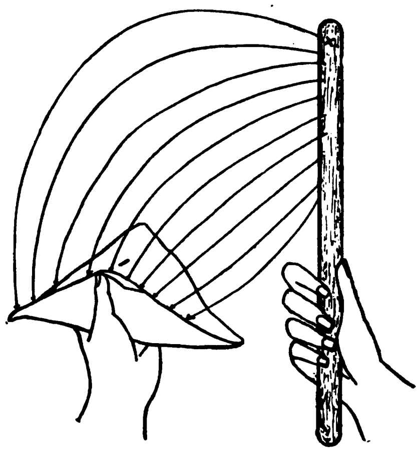


FIG. 65.

71. Electrostatic Field of Force. Induction.— The charged body has about it a field of force somewhat similar to the field about a magnet, but with some very important differences. The lines of electric force do not return to the body from which they start, but always end on another body having a charge of the opposite kind, as shown in Fig. 65.

Any body placed in a field of force will intercept the lines of force. Thus a ball between two oppositely charged rods will be the stopping place for lines leaving the + rod and the starting place for lines going to the — rod. It is, therefore, positively charged on one side and negatively charged on the other (see Fig. 66).

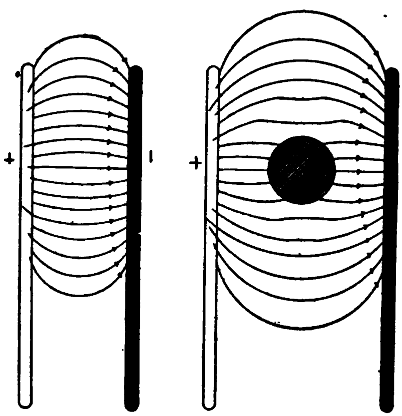


FIG. 66.

If there is a single charged body in a room the lines of force go to the walls of the room or to any object which may be near the charged body. Since every line has an end as well as a beginning, there is for every positive charge an equal negative charge somewhere, which is equivalent to saying that the sum of all the charges is zero. The force between two unit charges at 1 cm.

distance is 1 dyne; the force between charges q and q' at distance r is, in air:

$$(11) \quad f = \frac{qq'}{r^2}$$

In any other medium the force is

$$(12) \quad f = \frac{qq'}{Kr^2}$$

where K is a constant which must be determined for each substance. The constant is called the *specific inductive capacity.* It is a striking coincidence that the formulæ representing the force in the cases of gravitation, magnetism, and electrification should be identical. Let us see if we can explain the law in the last case by way of example.

The field of force at any point is measured by the number of lines of force to the sq. cm. at that point. The force at 1 cm. from unit charge is 1 dyne. At the same distance from a charge of 144 there are 144 lines to

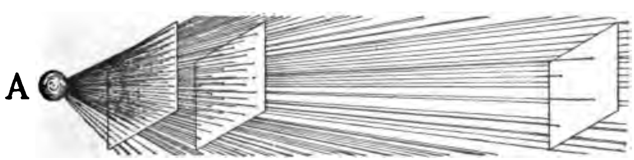


FIG. 67.

the sq. cm. (see Fig. 67). A body 1 cm. square would intercept 144 lines at 1 cm., but it would intercept only 36 at 2 cm. and 4 at 6 cm. For the lines are twice as

far apart at 2 cm. as at 1 cm., hence there are 6 rows of 6 each while at 6 cm. there are but 2 rows of 2 each. The stress in the field is therefore seen to diminish with the square of the distance. It is obvious that to double the charge at A would double the number of lines of force at all points of the field, or the attraction is proportional to q. The same is true of q', hence the force varies directly as the product of the charges and inversely as the square of the distance between them.

72. Nature of the Electric Charge. — Many, if not most, chemical elements consist of molecules containing two atoms which are alike except that one is electrically positive, while the other is negative. When these two atoms are united in a molecule they neutralize each other as far as any action outside the molecule is concerned, hence most bodies as we find them are not electrified. Let us suppose now that bodies differ among themselves in their affinities for the two kinds of atoms; thus wax has an affinity for negative atoms, while glass has an affinity for positive ones. The oxygen of the air is made up of such positive and negative atoms. When it touches wax the wax is not able to overcome the affinity of the negative oxygen atoms for the positive atoms with which they are united. If, however, a piece of wax and a piece of glass are brought together so close that the molecules of oxygen in the thin layer of air are very near to both substances at the same time, the plus atoms will be drawn in one direction and the minus atoms in the

opposite direction; their bond of union is overcome and the glass is coated with plus oxygen atoms, while the wax is coated with the same number of minus oxygen atoms (see Fig. 68). The charge of each atom being equal, the two charges are equal also. In our experiments we use for one of the bodies to be electrified a silk cloth rather than a bit of wax, because the cloth may be

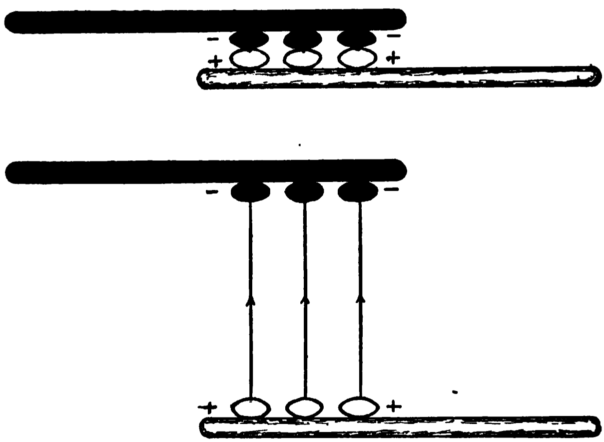


FIG. 68.

brought near to the glass at many places at once. We also rub the cloth along the rod to bring it in contact at as many points as possible.

The physical lines of force, according to the explanation just given, always begin upon a negative atom and end upon a positive atom. They may be thought of as chemical bonds stretched out to sensible distances.

73. **The Electrophorus.** — A metal plate filled with wax is rubbed with fur and thus negatively electrified. If, now, a metal plate provided with an insulating handle be brought near the wax (it may touch it at a few points without removing more than a small fraction of the

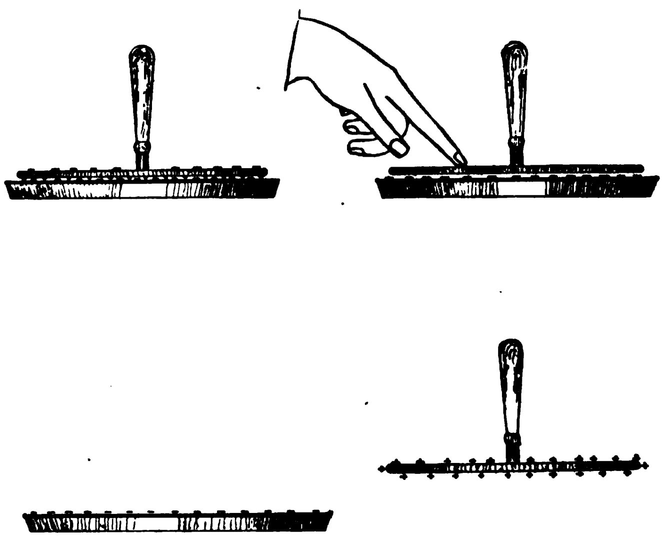


FIG. 69.

charge from so poor a conductor as wax), the plate will have a plus charge induced upon its lower surface (see Fig. 69) and an equal minus charge upon its upper surface. The lines of force, which start at the upper surface, end on the hand of the operator. If, now, the finger be touched to the plate the negative charge will

distribute itself over the body of the operator and thence to the earth, which is so large a conductor that the charge at any one place upon it is practically zero. Meanwhile the plus charge on the lower side of the plate is held there by the attraction of the minus charge on the wax, but if we take away the finger and lift the plate to a distance from the wax the plus charge will distribute itself over the plate and may be drawn from it in the form of a spark by presenting the knuckle to the edge of the plate. This operation may be repeated a large number of times. If the plate is allowed to give its charge to an insulated conductor, the latter may be charged.

The charge on the plate of the electrophorus was produced by *induction.*

74. Distribution of the Charge on a Conductor. — We have said that a charge distributes itself over a conductor. It does not follow that it always distributes itself uniformly, however. This is the case only when the conductor is a sphere. The farther the body departs from the spherical form, the more irregular is the distribution of the charge on the body. To express it differently: the charge has greatest intensity at those points where the curvature is greatest. This is analogous to the distribution of surface tension in a soap film and goes to support the theory that the charge consists of a strain in the ether at the surface of the conductor. It is evident that the charge on a plate

having rounded edges will be most intense at the edges, for there the curvature is greatest. If a rubber bag were stretched over a sphere the tension on the rubber would be alike at all points. If a cube of the same area were substituted for the sphere the tension would be greater at the edges than on the sides and would be greatest at the corners.

75. Electric Discharge. — Any elastic medium, if stretched beyond a certain point, gives way. The air which surrounds a charged body is in a state of strain. When a certain limit is reached the air gives way

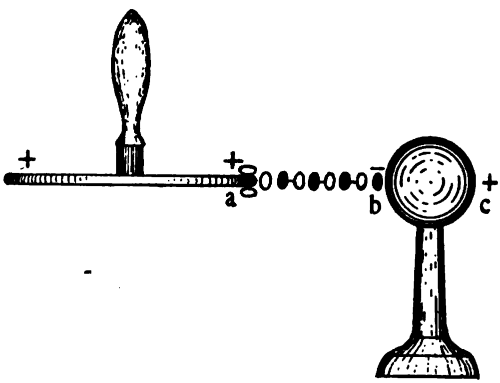


FIG. 70.

and the body is discharged: equilibrium is restored. When the edge of the charged plate of the electrophorus was brought near the sphere, a spark was seen and heard between the plate and the sphere: the plate was discharged. The air near the sphere must have been under greater strain than at other parts of the plate. Let us see why. The plus charge on the plate induced

a minus charge on the side of the sphere nearest to it (Fig. 70, *b*) and a plus charge on the opposite side. The minus charge on the sphere attracted the plus on the plate, causing it to accumulate at *a* until the attraction of the free plus atoms near the plate for the minus atoms of the oxygen in the air, assisted by the attraction of the minus atoms near the sphere for the plus oxygen atoms, became great enough to break down the bonds uniting the oxygen molecules in the space between. The result was to produce equilibrium between the sphere and plate, but not between these conductors and the earth. The plus charge at *c* still remains on the ball and a plus charge (less than the original charge by the amount on the sphere) remains on the plate. If the sphere had originally been connected to earth the plus charge would not have remained at *b* and the discharge would have resulted in perfect equilibrium. The union of the atoms of oxygen liberated heat just as the union of oxygen atoms with carbon atoms does in combustion. The rapid expansion of the air so suddenly heated produced a sound like that of a firecracker, when powder burns suddenly in a confined space. Thunder and lightning are produced by enormous discharges from cloud to cloud in the sky. When a highly charged cloud lies near the earth it may induce an opposite charge in the earth, resulting in a discharge between the cloud and the earth. The lightning strikes, as we say. When lightning strikes it follows the best conductor. The passage of a discharge through a tree

may heat the sap enough to vaporize it and rend the tree in splinters. The discharge from cloud to cloud evidently finds the shortest path not always the easiest. Fig. 71 is reproduced from a photograph taken at Tripp, South Dakota, July 22, 1898, by W. C. Gibbon.

FIG. 71.

76. Discharge from Points. — The curvature at a point is infinite; the intensity at a point is therefore so great that the charge escapes rapidly into the air. All conductors which are designed to hold a charge must have their edges rounded. The discharge from the points of leaves and blades of grass is usually suffi-

cient to equalize the potential between the earth and a thundercloud unless the cloud approaches very rapidly. This explains why lightning strokes are of such infrequent occurrence.

77. Electrical Machines. — It is often convenient to have at our command larger charges than can be obtained by rubbing a glass rod or by means of the elec-

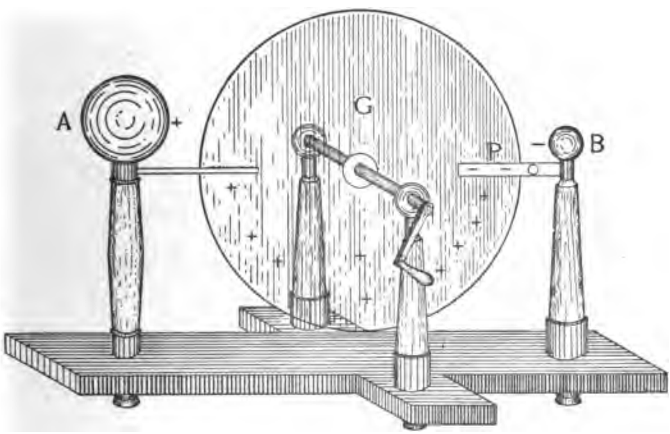


FIG. 72.

trophorus. Machines employing the principles of friction or of induction are known respectively as frictional or induction machines. An old-fashioned frictional machine is shown in Fig. 72. A large glass plate, *G*, is made to revolve between two leather pads, *P*. The plus charge on the glass is carried past the pointed conductor, where it draws off the induced negative charge, leaving

A positively charged. The negative charge passes from the pads to *B*, whence it may be conveyed to earth by a chain when only a positive charge is wanted. This machine has given place to a type of induction machines in which the inducing charge goes on increasing as the machine is operated. Such machines produce much more powerful charges than the frictional machine. The

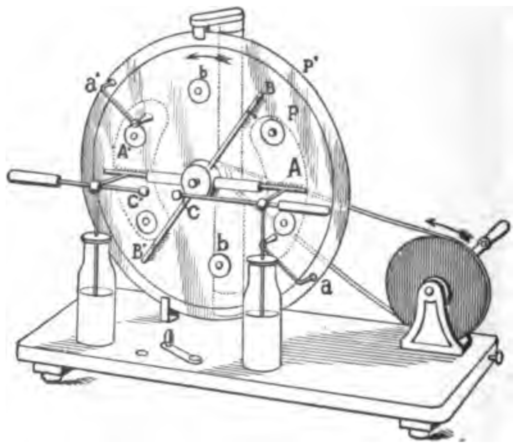


FIG. 73.

kinds most in use are the Toepler-Holtz machine and the Wimshurst machine. The Toepler-Holtz machine is shown in Fig. 73 and diagrammatically in Fig. 74. It consists of a revolving plate, *P*, which carries upon its front surface six or eight metallic buttons, *b*. Behind the revolving plate is a stationary plate, *P'*, which serves to support two paper armatures, *A*, *A'*, as they are called. In front of the revolving plate are an inducing

bar, BB', with points and brushes of wire at its ends, and two conductors, C, C', for receiving the charge. Two little arms, a, a', connected with the armatures, A, A', convey, by means of little metallic brushes, any charge

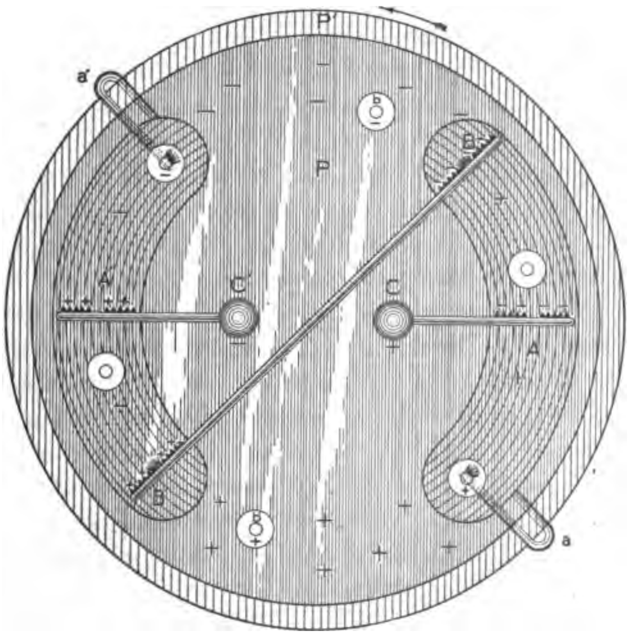


FIG. 74.

that is on the buttons, b, to the armatures. The action of the machine may be explained as follows: a small minus charge is produced on one of the paper armatures, say A', by rubbing it with flannel. This initial charge may be exceedingly small; indeed, if the machine has

been lately in use the small charge remaining on the armature will be sufficient. This small charge acts inductively through the glass on BB' to attract a plus charge to B' and repel a minus charge to B. These induced charges will escape from the points at B and B', electrifying the glass. Part of the charge, however, is given to the little buttons, b, which, as the plate rotates, carry it to the brushes on a, a', and thus to A, A'. The charges on the armatures are thus being continually augmented, and that, too, very rapidly, since the greater the charge on A, the greater the induction on B. Meanwhile the plus charge on the upper part of the glass plate, P, induces a minus charge on the points of C, which escapes, leaving C positively charged; and the minus charge on the lower part of P in like manner charges C' negatively. The charges C and C' continue to increase till the air between them gives way and a discharge occurs. The armatures, however, do not discharge, hence a second large charge on C and C' may be obtained quickly. Indeed, if C and C' are not too far apart, an almost continuous succession of sparks may be made to pass between them.

78. Potential. Capacity.—If a ball of 1 cm. radius receive unit charge it will be more intensely electrified than a ball of 10 cm. radius having the same charge, exactly as a given quantity of heat applied to the small ball will heat it hotter than an equal quantity applied to the large one. The term in electricity which corre-

sponds to temperature in heat is *potential.* The potential of a ball of 1 cm. radius having a charge of plus 1 unit is 1; the potential of the ball of 10 cm. radius having the same charge is 0.1. The *capacity* of a conductor is the quotient of its charge by its potential. The capacity of a body is known, then, if we can measure the charge which it receives, from a source of known potential. Potential is usually denoted by V. If we denote quantity of charge by Q and capacity by C we may write:

$$(13) \quad C = \frac{Q}{V}$$

The potential of the earth is reckoned as zero. The potential of all conductors in contact with each other soon becomes identical, just as the temperature of two bodies in contact soon becomes the same.

The potential of a conductor which is connected to the earth is therefore zero. The potential of the space surrounding a body diminishes as we leave the body. A charge always "flows" from places of high to places of low potential, just as heat flows from places of high to places of low temperature. A discharge which takes place through a good conductor, like a copper wire, is usually called a current. We are not to infer from the use of the term current that we are dealing with a fluid. What takes place in the wire seems to be different from what takes place in the air when a spark passes, but the result accomplished is the same: equilibrium or equality of potential is restored in both cases.

79. Condensers. — The potential of the space about a conductor is lowered by the presence of a conductor of lower potential. A plate, *A* (Fig. 75), connected to the positive conductor of an electrical machine, *C*, having a potential of 100 will itself have a potential of 100. If the capacity of the plate is 10 the plate will receive a charge of 1,000 units before its potential rises to 100. If we now hang near *A* a plate,

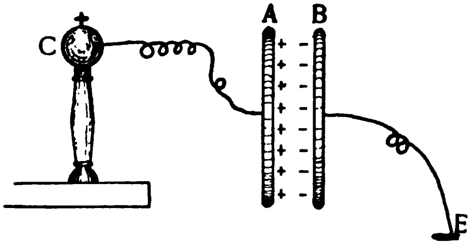


FIG. 75.

B, which is connected to the earth, the potential on the side of *A* next to *B* will be lowered and an additional quantity must flow from *C* before the potential of *A* will rise to 100. The closer *B* is to *A* the greater the charge required to equalize the potential.

Such an arrangement of plates as that just described is called a condenser because it allows a large charge to be collected in a small space, or, in other words, it increases the capacity of the conductor *A*. A common form of condenser is the Leyden jar. It consists of a glass jar coated to about two thirds its height with tinfoil, both outside and inside. A rod terminating in a ball passes through the insulating cover and communicates by a chain to the inner coating (see Fig. 76). The glass has a specific inductive capacity six times as great

as that of air, hence the capacity is six times as great as a condenser having plates of the same size. The jar is charged by connecting the ball to one pole of the electrical machine while the outer coating is held in the hand or otherwise connected to earth. It may be discharged by touching one end of a wire having rounded ends to the outer coating and then bringing the other end near the knob. A very powerful charge will sometimes pierce the jar. Franklin devised an experiment to prove that the charge is wholly in the dielectric and not at all in the conductor. A jar is made with removable coats (see Fig. 77). After being charged, the inner coating is removed by means of a glass rod or its ebonite handle, and the jar is lifted out of

FIG. 76.

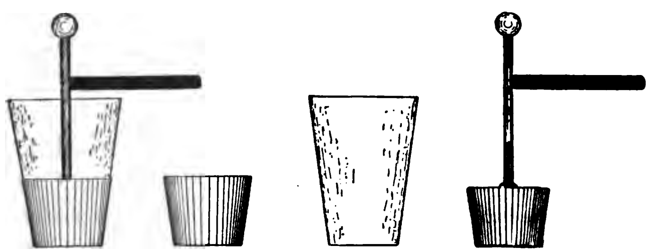


FIG. 77.

the outer coating. The coatings now show no signs of electrification; while the jar attracts pith balls. If the coatings are again put in place the jar will give a spark.

The purpose of the coatings is to distribute the charge to the glass in charging and to collect it at the moment of discharge. That the charge is not all given up by the glass is shown by the fact that it is usually possible to get a second (or residual) charge a few moments after a condenser has been discharged.

Electrical machines are usually provided with two Leyden jars. The positive conductor is connected with the inner coating of one jar, the negative with the inner coating of the other, and the two outer coatings being connected by a wire which may be broken by a switch when it is not desired to use the condensers (see Fig. 73).

80. The Gold-leaf Electroscope. — It is often desirable to make a more delicate test of electrical charges than can be made with the pith ball. For this purpose a gold-leaf electroscope is used. It consists of a rod rounded or furnished with a ball at its upper end and having attached to its lower end two strips of gold-foil. The rod passes through the stopper of a bottle which encloses the leaves, protecting them from draughts of air (see Fig. 78). When a glass rod is brought near the ball the electroscope has a positive charge induced in the leaves by induction. If while the rod is near the finger is touched to the knob the repelled plus charge will pass to earth, leaving the electroscope negatively charged. If we remove the finger and then the rod the electroscope will have a permanent charge. A nega-

tively charged body brought near will cause the leaves to diverge further, while a positively charged body will cause the leaves to collapse. If a rubber rod had been used instead of the glass rod the electroscope would have been positively charged. A convenient method for testing bodies for kind and amount of electrification is to use what is called a proof plane. It consists of a small metal button with round edges cemented to the end of a glass rod. A metal ball or button tied to a silk thread will do as well. The proof plane is touched to the body to be tested and the charge received by the plane is carried to the electroscope. If the charge is not strong the plane may be touched to the ball of the electroscope.

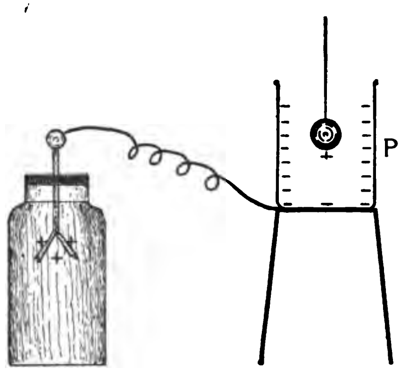


FIG. 78.

By means of an electroscope Faraday proved that the charge on a conductor induced by a charged body brought near it is equal to the charge on the inducing body. He performed the experiment with an ice-pail, hence it is known as Faraday's ice-pail experiment. It is an excellent illustration of the laws of induced charges. A small pail, *P* (Fig. 78), is supported on an

insulated support and a ball which has been charged by contact with a rubber rod is lowered by a silk thread into the pail. The leaves of the electroscope will diverge a certain amount. If the ball be now touched to the pail the charge on the ball will just neutralize the induced charge on the pail and the divergence of the leaves will not be in the least altered. This shows that the charge induced on the pail was exactly equal and opposite to the inducing charge on the ball.

81. The Discharge in Gases. — If we draw apart the knobs of an electrical machine while it is in operation a point will be reached at which sparks can no longer pass, but a hissing noise may be heard. If the room is darkened we may see faint brushes of purplish light issuing from one of the knobs. It will be found by testing that this is the negative conductor. Similar brush discharges may be seen on the plate and at the pointed conductors on various parts of the machine. The positively charged points show little dots of white light instead of the brushes. If we now connect the two knobs to wires which are sealed into the end of a glass tube which is connected to an air pump, it will be observed that when part of the air has been exhausted from the tube brush discharges will pass through the tube, and, if the exhaustion can be carried far enough, the whole tube will glow with pink light. Tubes may be obtained from which the air has been exhausted until not more than the thousandth part remains. They

are called Geissler's tubes. In such tubes the negative end glows with a violet light, while the positive end is pink, often arranged in layers or stratifications concave toward that end and reaching nearly to the negative end of the tube. A dark space separates the two discharges. The colors are not the same if other gases are in the tubes. The discharge in such exhausted tubes bears a close resemblance to the aurora borealis or northern lights, which are believed to consist of electrical discharges through the rare air in the upper regions of our atmosphere.

Tubes which are exhausted to a still greater degree than Geissler's are known as Crookes' tubes. The slight amount of gas remaining in a Crookes' tube shows no color, but the tube itself glows with a phosphorescent light opposite the negative end. The tubes are the source of Roentgen X-rays, which will be considered in another place.

Exercises.

52. (*a*) Place a bar magnet on a large sheet of paper. Set a small compass on the paper and make a dot on the paper near each end of the needle. Set the compass aside and connect the dots by a line. Repeat this operation for a large number of places on the paper. (*b*) Support a magnetized sewing needle by a thread fastened to it with wax, so that it will hang horizontally when no magnet is near it. Bring it over the centre of a bar magnet at a distance of say 10 cm. above the magnet, and then move it slowly toward one end of the magnet and afterward toward the other end.

53. A sailor was asked if he ever crossed the "line" (equator). He replied that he had. When asked how he knew, he

replied that he watched the compass and the instant the ship crossed the line the needle turned and pointed south. Explain.

54. An artesian well made by driving a long iron pipe into the ground was found to possess magnetic properties, such that knife blades thrust into the water as it flowed from the pipe were magnetized. Is it more probable that the water was magnetic or that the iron pipe was magnetized by being jarred while in the earth's field? If the point of a knife were rubbed on the upper end of this pipe, what sort of pole would the point become?

55. Why should care be taken never to drop or otherwise jar a magnet?

56. Why are magnets often made in a horseshoe form?

57. A knitting needle is bent into a horseshoe form and then magnetized. If the distance between its ends were measured before it was magnetized and again afterwards, would it be found greater before or after magnetization?

58. Would it probably have any effect on the time-keeping qualities of a watch if its hairspring should become magnetized?

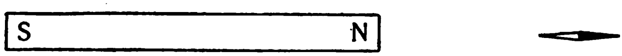


Fig. 79.

59. A long bar magnet has poles of strength 80. A compass needle having poles of strength 2 is pointing toward the N-pole of the magnet. The needle is 2 cm. long and its nearest pole is 4 cm. from the N-pole of the magnet (see Fig. 79). What is the force of attraction in dynes, what is the force of repulsion, and what the resultant force? What would the resultant force be if the distance were half as great?

60. If an iron rod be tapped with a hammer while it points in the direction of the dip needle it will become a magnet with the lower end a N-pole. If we now reverse it and tap it the

magnetism will be reversed, and the end which was at first down and is now up is a S-pole. In what position should it be held in order that it may be wholly demagnetized by tapping?

61. (*a*) Plot the field of a horseshoe magnet by means of iron filings. (*b*) In a similar way plot the field between two unlike poles of bar magnets at a distance of 8 cm. from each other and with a piece of soft iron 1 or 2 cm. square lying midway between them.

62. A 1-milligram weight lies on a glass plate and has a charge of 10 units upon it. How large a charge at a distance of 2 cm. will lift the weight from the glass?

63. Charge an insulated pail and test it inside and out with a proof plane and electroscope.

64. Put an electrical machine in motion and test the various parts for positive and negative electrification. Do not let the machine stop till the test is finished. Stop the machine, start again and repeat the tests to see if you get any indication that the machine reverses.

65. Why is it more difficult to brush lint from clothing in cold, dry weather than at other times?

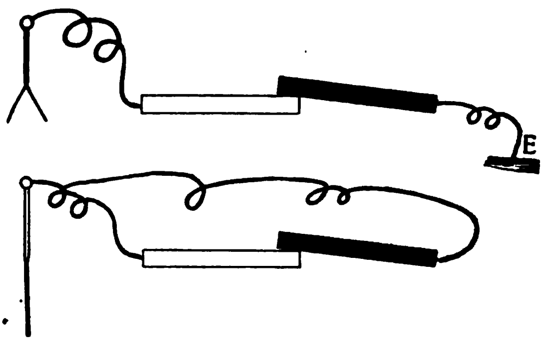
66. Lay a pane of glass on the table and rub it with a silk cloth. Dust a few fine iron filings on the glass. Lift the glass and note the behavior of the filings. Explain.

67. Rub a sheet of paper with a dry flannel cloth and place it against the wall of the room.

68. Let water from an elevated vessel flow out of a pipette at the lower end of a rubber tube. Bring an electrified rod near the water jet. Note and explain (*a*) the movements of the jet; (*b*) any changes in the jet itself.

69. Connect two metal handles by wires to the outside coatings of the jars of an electrical machine; arrange the knobs so that a short spark passes. By holding the handles observe the physiological effect of an electric current.

82. Electric Currents. — While electric charges are of great interest to us from the theoretical side, the greater part of the applications of electricity employ what we call *electric currents.* In 1786 Galvani observed that some freshly prepared frogs' legs lying near an electrical machine twitched when a discharge passed. In trying to use frogs' legs as an electroscope to test the electrification of the air the frogs' legs came in contact with an iron balcony and twitched when removed.



FIGS. 80 and 81.

Galvani believed this was due to a charge generated in the muscles of the specimen used, but Volta thought it could be explained by the contact of dissimilar metals. Volta constructed the first battery and from this time dates the study of electric currents. If a strip of zinc which is connected to ground be touched by a piece of copper which is connected to a very sensitive electroscope, the leaves of the electroscope will diverge, showing a difference of potential between the two plates

(see Fig. 80). This difference of potential by contact of dissimilar substances has been explained in Section 72. If we now connect the wire which was attached to the earth to the one attached to the electroscope the charge will disappear. A current has flowed through the wire from the copper to the zinc (see Fig. 81). The current lasts but an instant, however, like the discharge from a Leyden jar, only much weaker. If instead of bringing the metals in contact in air we bring both metals in contact with water, which has a salt or an acid dissolved in it, the current will flow continuously (see Fig. 82). Let us try to explain what occurs. The acid which is dissolved in water consists, let us say, of a positive atom of hydrogen, the chemical symbol for which is H, united to a negative atom of chlorine, the symbol for which is Cl. The molecules of acid in a solution are not like the molecules of oxygen in air, but a great many of the molecules are dissociated so that they pass quickly to the metals, the plus H going to the copper, while the minus Cl goes to the zinc. When the plates are joined outside the solution by a wire a discharge occurs through the wire, but other atoms of H rush to the copper plate and other atoms of Cl to the zinc plate, so that the current is kept up continuously. The H atoms unite with each other and form hydrogen bubbles which rise to the surface of the liquid, while the chlorine combines with the zinc.

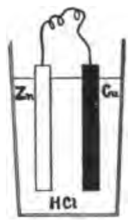


FIG. 82.

83. Polarization. — The chief difficulty experienced in the use of batteries grows out of the fact that the bubbles of gas formed on the plates do not quickly leave the plates, but form a layer of gas over them, which prevents the dissociated atoms from reaching the plates and so weakens the current and finally causes it to cease almost entirely. This phenomenon is known as *polarization*, and the chief differences between different sorts of batteries lie in the various devices used to get rid of polarization.

84. Magnetic Effect of Currents. — Oersted discovered in 1819 that a wire carrying a current will deflect a magnetic needle. If the wire is held parallel to the needle the needle tends to turn at right angles to the wire, the amount of deflection of the needle depending on the strength of the wire. A needle which is suspended on a horizontal axis will, if placed successively in different positions about a wire which is carrying a current, take the directions shown in Fig. 83. The central spot represents a section of the wire. Since the needle always tends to set itself parallel to the lines of force in the field, there must be in the space which surrounds a conductor carrying a current a field of magnetic force in which the lines of force are circles enclosing the conductor.

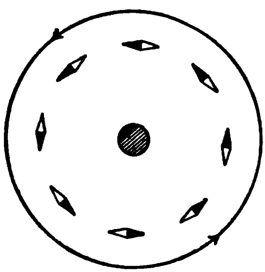


Fig. 83.

A compass set in a block of wood with a wire running beneath it makes a simple *galvanoscope.* If the wire carrying a current be connected so that the current flows from north to south through the galvanoscope, the needle deflects toward the west at its north end (see Fig. 84). If the connections be changed so that the current flows the other way, the needle will deflect to the east.

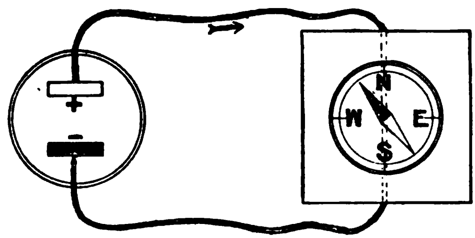


FIG. 84.

If we bend the wire back over the compass so that the current flows north above the needle, but south below it, the deflection will be increased, since each part of the wire deflects it in the same direction (see Fig. 85). By winding the wire a number of times about the needle the effect is correspondingly increased and the instrument becomes sensitive enough to detect currents of very small intensity.

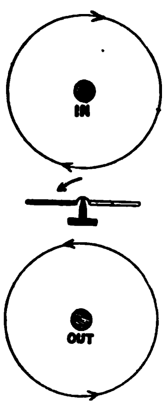


FIG. 85.

85. Batteries. — A battery consists essentially of two unlike conductors immersed in a solution. One of the conductors is a metal, the other sometimes a metal,

sometimes carbon, carbon being almost the only solid substance outside of the metals which is a good conductor. We have seen that some plan must be used to get rid of the products of the chemical change which occurs in the battery and so avoid polarization. The battery most commonly used is the Daniell's cell in the form known as the gravity cell, because its two fluids are kept apart by their difference in specific gravity (see Fig. 86). We may begin with a cell having a copper plate at the bottom, a zinc plate at the top, and a very dilute solution of sulphuric acid. Sulphuric acid consists of two radicals or combinations of atoms which are treated like atoms, namely, hydrogen and sulphion, H_2 and SO_4. When the plates are connected by a wire the hydrogen collects on the copper plate and soon polarizes the battery. To prevent this action the copper plate is covered with copper sulphate, $CuSO_4$. Now sulphion, as the radical SO_4 is called, has a stronger affinity for hydrogen than for copper, so it gives up its copper atom, which is deposited on the copper plate, and unites with the hydrogen to form sulphuric acid. The chemist expresses this reaction by an equation thus:

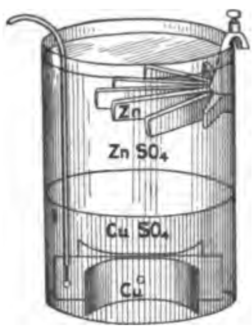


FIG. 86.

$$H_2 + CuSO_4 = Cu + H_2SO_4$$

But sulphion has a stronger affinity for zinc than for hydrogen, so it gives up its hydrogen and takes zinc, making zinc sulphate, $ZnSO_4$:

$$Zn + H_2SO_4 = H_2 + ZnSO_4$$

and so the merry round keeps up and the difference of potential remains constant while a constant current flows through the wire. The battery thus consumes zinc and copper sulphate, while it produces copper and zinc sulphate. The zinc sulphate must occasionally be siphoned off and replaced by water, while copper sulphate crystals are added to make good the loss. This is the only battery which works best if kept in constant action. A gravity or Daniell's cell should be kept on closed circuit when not in use; all other batteries should have the circuit broken instantly when they are not to be longer used.

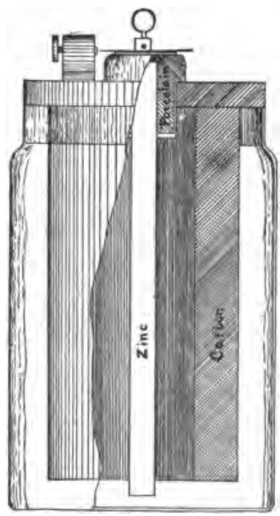


FIG. 87.

While the current furnished by the gravity cell is steady, it is not strong. The various carbon batteries, known as Leclanché or sal ammoniac batteries, give a strong current for a short time, but soon polarize. They recover quickly, however, and so are well adapted

to use for electric bells and telephones. Usually a zinc rod, a large carbon plate, and a solution of ammonium chloride (sal ammoniac) constitute the battery (see Fig. 87). The bubbles of hydrogen soon escape from the large rough surface of the carbon. This battery is cheap and easily renewed.

When a strong current for a considerable time is wanted the plunge battery or chromic acid battery is used. In this battery a large zinc plate is hung from the lower side of a piece of vulcanite between two carbon plates. The plates are plunged into a solution of chromic acid. The plates must be lifted whenever the battery is not in use. This battery requires frequent cleaning and renewal, but it is the best resort for strong currents when storage batteries or dynamo currents are not to be had.

86. Chemical Effects of the Current. Electrolysis. — The direction taken by the dissociated atoms in a solution with two unlike plates dipping in it is determined by the nature of the plates. If two like plates dip in such a solution there will be nothing to determine the direction, and consequently no effect will be observed. Now suppose two plates of platinum, which

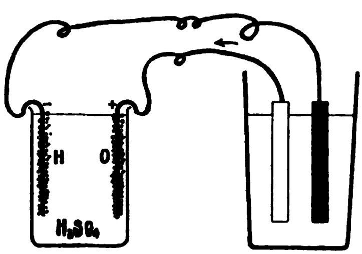


FIG. 88.

dip in dilute sulphuric acid, are connected to the plates of a battery, as shown in Fig. 88. Atoms of hydrogen at once go to the negative plate, the sulphion takes hydrogen from the water molecules (H_2O), leaving free oxygen, which collects on the positive plate. The final result is to decompose the water into hydrogen and oxygen. If the plates are held under test tubes the gases may be collected, when the volume of hydrogen will be found just twice that of the oxygen.

If lead plates had been used the oxygen would have united with the lead, forming lead oxide. The cell, with two unlike plates, lead and lead oxide, would, when disconnected from the battery, be itself a battery. This, somewhat modified, is the principle of the lead storage battery.

87. Electroplating. — An important application of electrolysis to the arts is in the deposition of metals by the electric current. The object to be coated is made the negative terminal (attached to the zinc plate) of a battery, while the positive terminal is composed of a plate of the metal to be deposited. Both plates dip in a solution of a salt of the metal to be deposited. The current deposits the metal on the negative terminal, or electrode, as it is called, while the metal from the positive electrode goes into the solution.

It is sometimes easiest to separate metals from the ores which contain them by chemically producing the salts of the metals. The metals may then be recovered

from the salts by electrolysis. Copper is produced on a large scale in this way.

88. Electrotyping. — Books are now seldom printed from the movable type in which the matter was at first set. A mould is made from the type in wax, which is then coated with finely powdered graphite (a form of carbon) to make it conduct, and is made the negative electrode in a bath of copper sulphate. The current deposits copper in the mould in a form which perfectly reproduces the type. The thin sheet of copper is backed by type metal and wood of the proper thickness and the types are free to be used again without having been worn in the least, while the plates, after use, may be laid away till another edition of the book is wanted.

89. Heat Produced by the Current. Resistance. — When one body moves upon another the rubbing of the molecules against each other sets them in motion, or, we say, friction produces heat. Something analogous to friction, but of course very different, hinders the transfer of an electric charge. We have seen that the air is intensely heated in the path of a spark discharge. A wire offers much less resistance to the passage of a current than air does, but the current always heats the wire and the fluid of the battery also. If the fluid is a good conductor, and a bad conductor is in the circuit outside the battery, the battery will be heated but little, while if the battery is a poor conductor and the circuit

is a good one, the battery will become heated more than the wire. The resistance of a conductor is the exact opposite of conductivity. If we indicate conductivity by C and resistance by R we may write:

$$C = \frac{1}{R} \text{ or } R = \frac{1}{C}$$

Ohm proved that the resistance of a conductor of uniform cross section varies directly as its length and inversely as its cross section. Two wires of the same length and diameter have different resistances if they are of different materials; that is to say, substances have specific resistance (or conductivity) just as they have specific density and specific heat. The resistance of a conductor whose uniform cross section is a, whose length is l, and whose specific resistance is r, is:

$$(14) \quad R = \frac{rl}{a}$$

If the cross section is a circle the area of the cross section is 3.1416 times the square of the radius. The cross section of two wires of equal diameter is evidently twice as great as that of a single wire. Two wires side by side have, therefore, but half as much resistance as one wire. The resistance of a pure metal varies greatly with its temperature, increasing as the temperature increases. There is good reason to believe that at absolute zero all pure metals would conduct perfectly. Certain alloys have nearly the same resistance for a considerable range of temperature. German silver is

an alloy of this sort, which is much used in making standards of resistance.

90. Ohm's Law. Units.—We have learned that a current flows because of a difference of potential, and that the intensity of the current is proportional to that difference of potential. We have further seen that the intensity of the current is diminished as the resistance of the conductor increases. Ohm wrote:

$$(15) \quad i = \frac{E}{R}$$

where i denotes intensity of current (or current simply), E denotes difference of potential (sometimes called electro-motive force), and R denotes the total resistance of the circuit. It follows that:

$$(15a) \quad E = Ri \text{ and } (15b) \quad R = \frac{E}{i}.$$

The units in everyday use for measuring these quantities are named for three men, who laid the foundation for our knowledge of electric currents. Intensity of current, i, is measured in *ampères*, difference of potential, E, is measured in *volts*, while resistance, R, is measured in *ohms*:

$$\textit{ampères} = \frac{\textit{volts}}{\textit{ohms}}$$

An ohm is the resistance of a thread of mercury 1 mm. in cross section and 106 cm. long. A volt is the difference of potential between the terminals of a gravity cell. An ampère is the current which would flow through a

resistance of one ohm with a difference of potential of one volt. More exact definitions than these may be given, but these will answer all practical purposes.

We shall get some notion of the magnitude of the units by employing them in various ways.

91. **Useful Heat Effects.** — By using good conductors in a circuit, except at particular points which have a high resistance, we may obtain intense heat at those points. In the two forms of electric lighting in common use this is done. The arc light consists of two rods of carbon the ends of which are allowed to touch, closing the circuit. The resistance at the point of contact (the contact being poor) is high, the carbons become heated, and are then drawn a short distance apart. The current continues to pass the gap, following the heated carbon vapor; a small hollow or crater is formed in the positive carbon, which is heated white hot and furnishes most of the light. The positive carbon is usually placed uppermost, so that the light from the crater may be thrown downward (see Fig. 89). The temperature of the arc is estimated at 4,000° C. It is the highest temperature which can be obtained artificially, and is sufficient to volatilize all substances. The light is by far the most intense of any artificial light known. The arc lamp requires about 6 or 8 ampères at 50 volts and gives 1,000 candle power. Various mechanical and electrical devices are used to open the arc after the current has started and to feed the carbons

together as they waste away. Arc lamps are usually connected in series, so that the total resistance of the line is the sum of the resistances of all the lamps (see Fig. 90). A line of twenty lamps would have a resistance of something like 125 ohms and would require a difference of potential of 1,000 volts. As the negative terminal of the dynamo and the farther end of the line are usually connected to ground, it is not safe for a person standing on the ground or on a wet floor to touch such a circuit.

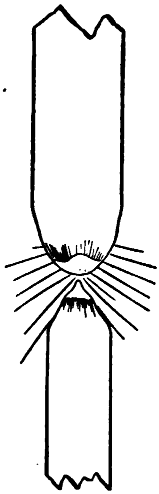


FIG. 89.

For lighting houses the incandescent lamp is used. It consists of a thread or filament of carbon, often a carbonized silk thread, sealed within a glass bulb and connected with the circuit by platinum wires, which are sealed into the glass. The air is exhausted from the bulb so that the carbon, when heated white hot by the current, is not burned.

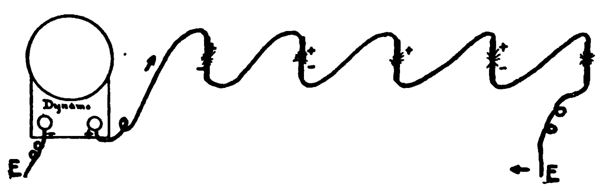


FIG. 90.

The common size of incandescent lamp gives 16 candle power and uses 0.5 ampère at 100 volts, or 1 ampère

at 50 volts. Incandescent lamps are connected parallel, the two line wires being kept at a constant difference of potential (see Fig. 91). The more lamps there are connected at any one time, the more current flows, or, in other words, the less the resistance of the circuit. A lamp is turned on by closing a gap in the circuit by means of a key. Electric lamps give off no smoke and consume no oxygen from the air. They must, however, be used full power or not at all, as commonly made, and are more expensive than the so-called incandescent gas lamp invented by Welsbach for the same illuminating power. The ability to turn the lamp on or off at a moment's notice and to control a number of lamps in a ceiling or other inaccessible place from a single convenient point is a great convenience. Moreover, the incandescent lamp is perfectly cleanly and requires no attention whatever.

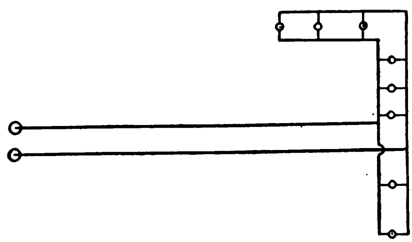


FIG. 91.

The electric heating of houses and cooking by the heat from the current, while perfectly possible and very convenient, have not become common because too expensive.

The welding of metals by the electric current is accomplished by connecting the metals in the circuit as the carbons of an arc lamp are connected. The heat

generated at the point of contact increases the resistance at that point, which still further heats the metal until it is soft enough to unite, when the ends are pushed together and the current is cut off. A very large current of low potential is required for this work.

92. **The Current Produced by Heat.** — The contact difference between two metals in air is different for different temperatures. If two strips of unlike metals are connected to form a closed circuit, there will be no current flowing in the circuit as long as the two places of

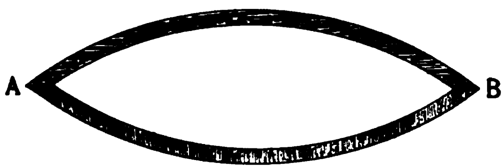


FIG. 92.

union (*A*, *B*, Fig. 92) are at the same temperature, since the differences of potential at *A* and *B* are the same; but if *A* be placed near a flame while *B* is kept cool, the differences of potential at *A* and *B* will not be the same, and a current will flow through the circuit so long as the points *A* and *B* are kept at different temperatures. A series of such elements is shown in Fig. 93. A series of 50 or 100 such elements made of antimony and bismuth and joined in series with a delicate galvanometer make an exceedingly sensitive thermometer.

Thermo-electric generators are now made which take

the place of batteries, the heat from a single Bunsen burner furnishing a difference of potential of 5 volts and a current on short circuit of 4 ampères. Such a generator serves admirably for charging storage batteries and for electroplating and the running of small motors. The difference of potential is very constant.

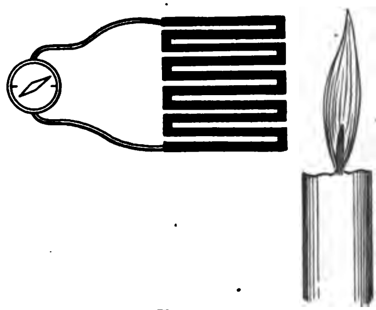


FIG. 93.

93. Motion Produced by the Current. — We have seen that a magnetic needle is deflected by the passage of a current in its neighborhood because the conductor is surrounded by a field of magnetic force. Two parallel currents will attract or repel each other, depending on

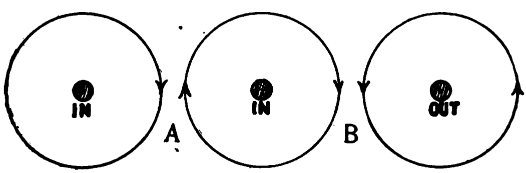


FIG. 94.

their direction with reference to each other. If going in the same direction they will attract, if going in opposite directions they will repel. The lines of force always tend to neutralize each other just as electric charges do, or as the two opposite stresses at the two

ends of a stretched cord do. At *A* (Fig. 94), where the currents are both flowing into the paper, the magnetic lines are seen to be opposite and the currents attract; at *B*, where the currents flow in opposite directions, the magnetic lines between the wires, being alike, repel.

94. Solenoids. Electro-magnets. — A long wire wound spirally about a cylinder has, when carrying a current, a field of magnetic force about it which is identical with the field of a bar magnet (see Fig. 95).

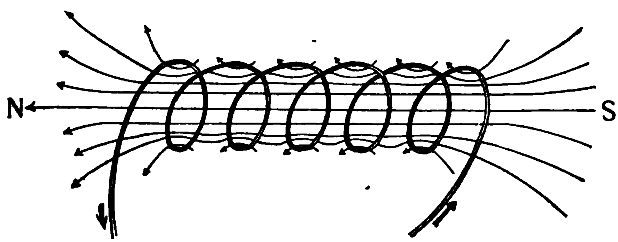


FIG. 95.

Two such solenoids behave toward each other like two bar magnets. If a core of soft iron is placed in the solenoid the field of force becomes very much stronger; indeed, such an electro-magnet, when carrying even a moderately strong current, is more powerful than a good bar magnet. Its chief usefulness depends upon the fact that its strength may be increased or diminished at will by increasing or diminishing the current, the magnetism disappearing almost wholly the instant the circuit is broken. Electro-magnets are oftenest made in the horseshoe form to give them great lifting power or

to make the path of the lines of force as much in the iron as possible, rather than in air, where they are less intense.

95. The Electro-magnetic Telegraph. — An electro-magnet having a piece of soft iron which is supported by a hinged bar and held away from the magnet by a spring constitutes what is known as a sounder. The movable piece of iron is called the armature. The sounder is the instrument used for receiving messages, the sending being done with a simple switch called a

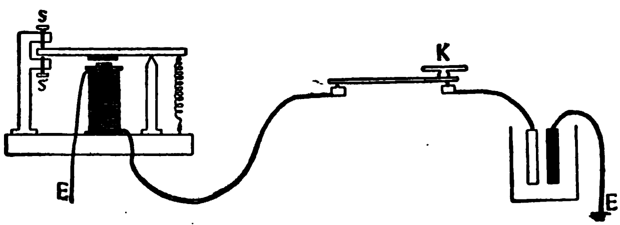


FIG. 96.

key. Let the key, *K*, be open, the spring holds the armature up against the stop, *s*. Closing the key allows the battery current to flow through the electro-magnet, the armature is drawn down, strikes the stop, *s′*, and makes a sound. When the key is opened the magnet no longer acts and the spring draws the armature back against *s*. In Morse's printing telegraph, which is now but little used, a point on the stop made a dot on a moving paper if the circuit was kept closed a very short time, or a dash if the key was kept closed a longer time. Although the messages are now read

almost universally by ear, the signals are called dots and dashes, and various combinations of dots and dashes constitute the alphabet. The code of signals is as follows:

a —	*k* — - —	*u* - - —	1 - — — -
b — - - -	*l* ——	*v* - - - —	2 - - — - -
c - - -	*m* — —	*w* - — —	3 - - - — -
d — - -	*n* — -	*x* - — - -	4 - - - - —
e -	*o* - -	*y* - - - -	5 — — —
f - — -	*p* - - - - -	*z* - - - -	6 - - - - - -
g — — -	*q* - - — -	*&* - - - -	7 — — - -
h - - - -	*r* - - -	, - — - —	8 — - - - -
i - -	*s* - - -	? — - - — -	9 — - - —
j — - — -	*t* —	. - - — — - -	0 ———

It is evident that by closing *K* a signal will be made at the sounder, no matter how far away *K* is placed, provided that the battery is strong enough to send a current of sufficient strength through the line. The resistance of a long telegraph line is several thousand ohms, and the currents are weak even when the return circuit is made through the earth.

For operating on long lines a device called a *relay* is used. It consists of an electro-magnet having a large number of turns, small wire being used so as to bring the current close to the iron core. The armature of a relay is hinged to move with a small force so that a weak current can actuate it. This armature when drawn toward the magnet acts as a key to close a local circuit, having a battery and sounder. The local circuit has a low resistance, and is readily operated by a few cells with force

enough to give as loud a sound as is desired. A line containing a relay and local circuit is shown in Fig. 97.

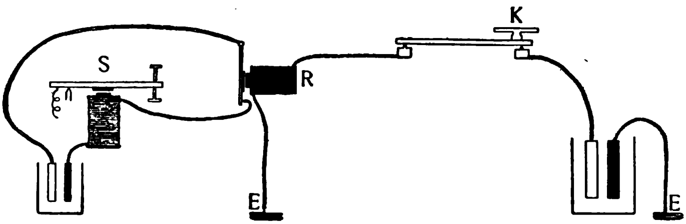


FIG. 97.

All of the instruments are duplicated at every office on the line. Every key has a switch, which is kept closed except when the operator is sending a message. Each office has a particular letter by which it is called. Every office on the line may read any message which goes over the line. When a particular office is wanted its call is repeated till the operator answers by opening his switch, after which he closes the switch and receives the message.

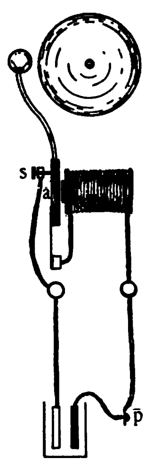


FIG. 98.

96. Electric Bells. — The electric bell is arranged to break its own circuit as fast as it is closed, thus keeping up a succession of taps as long as the key or push button is kept closed. Its connections are shown in Fig. 98. When the circuit is closed by pushing the button at *p*, the armature, *a*, is drawn toward the magnet and the hammer strikes the bell. By mov-

ing away from the stop, *s*, the armature breaks the circuit, the spring draws the armature back, and the cycle of operations is repeated.

97. Electric Motors. — An electric motor consists of an electro-magnet in the field of which another electro-magnet is made to revolve continuously by making and

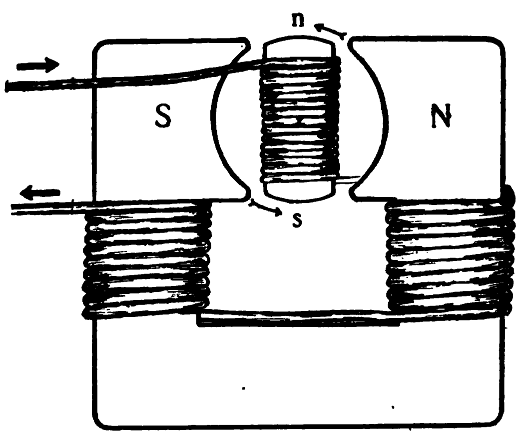


FIG. 99.

breaking the circuit through it. The stationary magnet is called the field magnet, the movable one the armature. It is evident that if a current is made to flow through *ns* (see Fig. 99) when it is in the vertical position, it will rotate in the direction of the arrow. Now if, when *ns* has reached the horizontal position, the current in it be reversed, thus reversing the poles in

the armature, the latter will continue to rotate in the same direction. The armature is mounted on a shaft which rotates between the poles of the field magnet and the reversal of the current is accomplished by means of a device called a commutator (see Fig. 100). The commutator consists of a divided ring fastened to the shaft and insulated from it. The ends of the wire which form the coils of the armature are connected to the two halves or segments of the commutator. Two strips of copper or carbon called brushes are supported in a fixed position so as to rub upon the rotating commutator. The wires carrying the circuit are attached to these brushes. It is evident that as the armature rotates, the direction of the current in the armature changes once during each revolution. The motion will be more steady if the armature have two coils at right angles to each other. The commutator must then have four segments and each coil will be idle half the time (see Fig. 101). In large motors the commutator often has 120 or more segments. A typical motor is shown in Fig. 102.

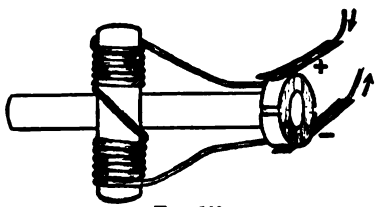


FIG. 100.

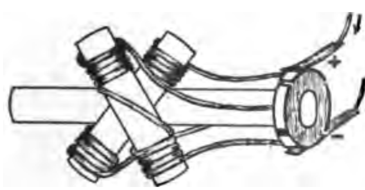


FIG. 101.

98. Currents Produced by Motion. — We have seen that currents may be the result of chemical action and that chemical action may be produced by a current, likewise that currents may be produced by heat and that heat is an effect easily produced by the current. It is perhaps to be expected, then, that since motion may be produced by a current, a current might be produced by motion. Such is, indeed, the case whenever a conductor is moved across the lines of magnetic force. This fact, which was discovered by Faraday in 1837, is of the greatest importance. Such currents are known as induced currents. The laws which govern their production may easily be illustrated by means of a coil of wire attached to a galvanoscope, a bar magnet, and an electro-magnet having a removable core. The coil attached to the galvanoscope is usually referred to as the secondary circuit (see Fig. 103). The electro-magnet is called the primary circuit. The galvanoscope should be far enough removed so that its needle will not be much disturbed by the movements of the magnet. If we thrust a bar magnet into the coil the needle of

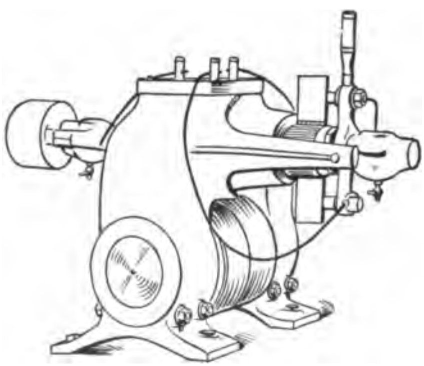


FIG. 102.

the galvanoscope will be deflected, but when the magnet comes to rest the needle will return to zero. Now if we quickly pull the magnet out, the needle will be deflected in the opposite direction. If the other pole of the magnet be used the deflection will be opposite in direction. The primary coil, which is a magnet when a current flows through it, will produce exactly the same effects, the deflections being greater when the core is in the coil. If the primary coil be placed in the secondary coil while connected with the battery and a variable resistance, currents will be induced

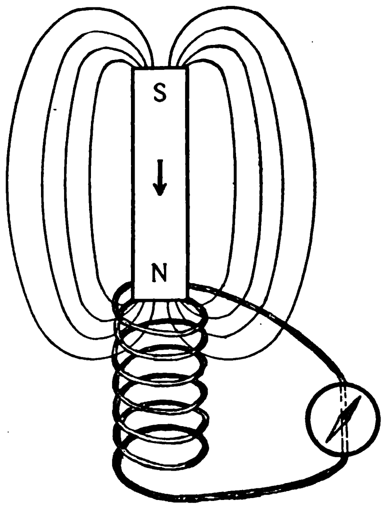


FIG. 103.

if we suddenly increase or diminish the resistance. Similarly, if we break or make the primary circuit, currents will be induced in the secondary.

The quickness with which the change is made in all the cases mentioned determines the difference of potential produced. It will thus appear that we are not limited as we are in the case of batteries and thermopiles, but may produce very high potentials without

very high resistances. All that is required is that there shall be a conductor moving rapidly in a field of magnetic force, or, better, a field of force rapidly changing in intensity in the neighborhood of the conductor.

99. Spark Coils. — A primary circuit having an interrupter like that of an electric bell will, if it consist of a large number of coils, induce at the instant the circuit is broken a difference of potential in itself which opposes the breaking of the circuit and produces a spark. In this case each coil of the circuit acts upon its neighbors and the process is called self-induction.

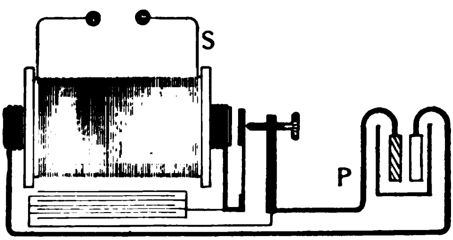


FIG. 104.

Such simple spark coils are used for lighting gas and to a limited extent for medical purposes. The common form of induction coil (Ruhmkorff's, Fig. 104) has but few turns in the primary coil, while the secondary, which is around it, has a very large number of coils of fine wire. Such coils are used to illuminate vacuum tubes and for medical purposes. The terminals of the primary are usually connected to a condenser which reduces the self-induced spark in the primary. This primary spark is a disadvantage in two ways: it burns the contacts and it diminishes the spark in the secondary

by retarding the break in the primary. The spark produced by an induction coil is identical in its effect with that produced by the Toepler-Holtz machine.

100. The Dynamo. — Any electric motor if connected to a galvanometer and turned by hand will show a current through the galvanometer. Indeed, the conditions for generating a strong continuous current are exactly met in the motor. Such a machine when used for producing a current is called a dynamo-electric machine or, more briefly, a dynamo. The construction of the machine we need not describe again. It is identical with the motor. The field magnets have a small amount of permanent magnetism — sufficient to start a small current in the armature coil as soon as it begins to rotate. This current, or part of it, flows through the coils of the field magnet and the potential soon rises to a maximum and remains steady for a constant speed and load. In order to explain the induced current in terms of the

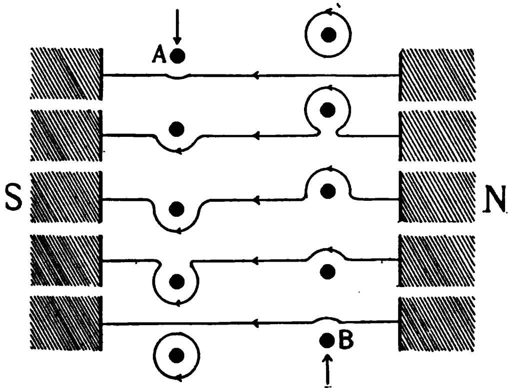


Fig. 105.

magnetic field we must now recall that a magnetic field is thought of as filled with elastic lines of force which tend to contract lengthwise and mutually repel. A wire carrying a current has lines of force about it. If these lines of force about the wire are more numerous in one part of a circuit than in another their mutual repulsion causes them to spread out and equalize the potential. How does a wire which moves across lines of force get the lines wrapped about it — that is, get a current flowing through it? Let the row of black dots in Fig. 105 represent the successive positions of a wire which is cutting through a line of force as it moves downward through the field. The line stretches more and more till it finally breaks, reuniting above and leaving a line of force about the wire in the direction corresponding to the lines about a current which is flowing into the paper. At *B* a wire moving upward

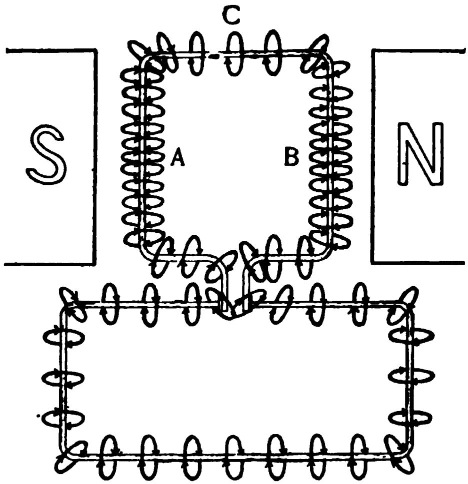


FIG. 106.

would have lines about it corresponding to a current out of the paper. If *A* and *B* are the upper and lower sides of a coil then the current flows round the coil. The lines (rings) of force which are constantly forming at *A* and *B* are as constantly spreading out along the wire to equalize the potential. See a top view in Fig. 106, where it may be seen that the lines of force which

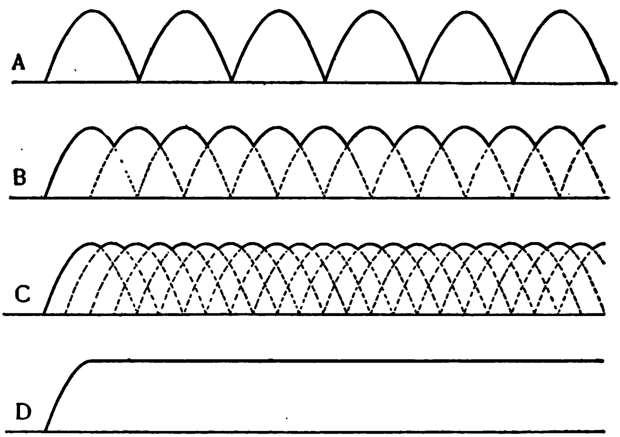


FIG. 107.

reach *C* and *D* from the two points *A* and *B* are going in the same direction and hence the current flows in one direction in all parts of the wire. When *A* is at the top and *B* is at the bottom they are moving parallel to the lines of force and cut none of them. They therefore generate no current at this part of the revolution, while a maximum current is generated when the coil is cutting squarely across the lines. When *A* be-

gins to ascend and *B* to descend the current will be reversed in the coil, but owing to the commutator it will always flow in one direction through the external circuit at *D*. It will be seen that a dynamo having but one coil and but one pair of segments in its commutator would give a very unsteady current. Increasing the number of coils increases the steadiness of the current. In Fig. 107 the current strength for successive short intervals during three revolutions of the dynamo is shown by the curves. At *A* the current falls to zero twice during each revolution. At *B* with two coils it falls but .7 as low, at *C* with 4 coils but .9 as low, and at *D*, where 24 coils are used, the current does not vary to any perceptible extent. A modern dynamo is shown in Fig. 108.

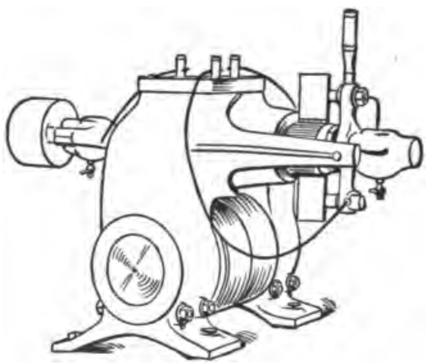


FIG. 108.

101. Alternators. — For certain purposes an alternating current has advantages over a direct current. If we solder the ends of our armature coil to a pair of rings which are not split, but placed side by side on the shaft and insulated from the shaft and from each other, the

brushes which rest on these rings will convey to the circuit the identical current which flows in the armature coil, that is, an alternating current. It is desirable that the alternations occur very frequently. The field magnet is therefore usually made with a number of poles wound so that alternate poles are north poles. The coil then passes from a north to a south pole sev-

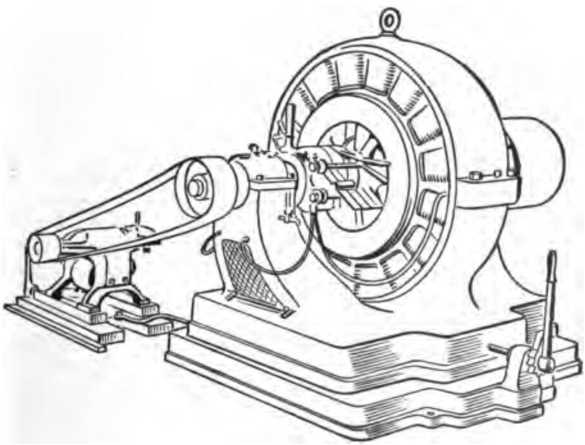


FIG. 109.

eral times during each revolution. The field magnets cannot be maintained by the alternating current, as they would be continually changing polarity. It is usual to employ the current from a small direct current dynamo for the purpose. Fig. 109 shows an alternating current dynamo with the accompanying direct current exciter driven by a belt from the shaft of the alternator.

102. The Transformer. — If the current from an alternator is sent through the primary coil of an induction coil while the hammer is wedged shut fast, an induced alternating current of much higher potential but smaller in amount will flow through the secondary. If the current from the alternator is sent through the long secondary coil a large current of low potential will flow in the short coil of the primary. It is very desirable in the distribution of electric currents over large towns to use high potentials, since this makes possible the use of small wires for conveying the currents. These high potential currents are not safe for common people to handle, but by sending them through a transformer they may be changed to any desired potential. The transformer consists of an iron core around one part of which are wound a large number of turns of fine wire, which carries the high potential current, *P*, from the dynamo (Fig. 110). Around the other part are wound a few turns of large wire, *S*, which carry a low

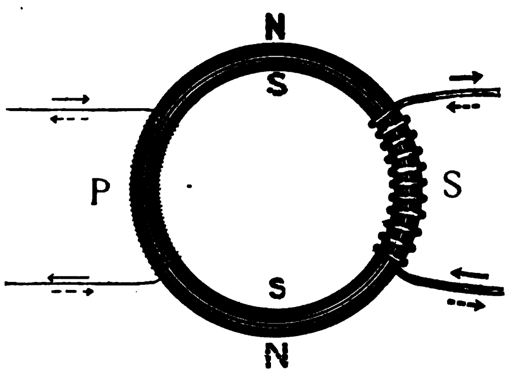


FIG. 110.

potential current to the house. Such currents are exactly as good for use in incandescent lamps as direct currents; for, although the current falls to zero at every alternation, the alternations occur so often (more than a hundred times per second) that the filament of the lamp has no time to fall in temperature between times.

Exercises.

70. Connect the zinc and copper plates of a gravity cell to a galvanoscope, but use water with a few c.c. of sulphuric acid instead of the regular copper sulphate solution. The instrument must be sensitive enough to deflect at least twenty degrees. Note the changes in deflection for half an hour. Now remove about half of the acidulated water and replace it with copper sulphate solution and add a small handful of copper sulphate crystals. Note the deflections for half an hour. Test the same battery with the same instrument every day for a week, keeping the circuit closed between times through some resistance like a telegraph sounder.

71. (*a*) Connect through a galvanoscope or an ammeter each of two gravity cells separately, then the two in series, then the two parallel. (*b*) Compute what current you ought to get in each case if the cells had each a potential difference of 1 volt and a resistance of 5 ohms. What is the sum of the potential differences in each case? of the resistances?

72. What is the effect on the potential difference of a cell of: (*a*) doubling the size of the plates, (*b*) moving the plates nearer together, (*c*) effect on resistance of doubling the size, (*d*) effect on resistance of bringing the plates nearer together?

73. Six gravity cells, each having a potential difference of 1 volt and a resistance of 5 ohms, may be connected in four different ways as shown in Fig. 111. Which is the best way to connect them when the largest obtainable current is desired:

(*a*) through a low resistance, say 1 ohm, (*b*) through 7 ohms, (*c*) through 40 ohms?

74. What sort of battery would best be selected (*a*) for electric bell work, (*b*) for an induction coil, (*c*) for telegraphy?

75. Two metals, bismuth and antimony, have a contact difference of potential which is changed .000117 volt for 1° C. How many bismuth-antimony pairs must be connected in series to give a potential difference of 1 volt if the temperature difference is 100° C.?

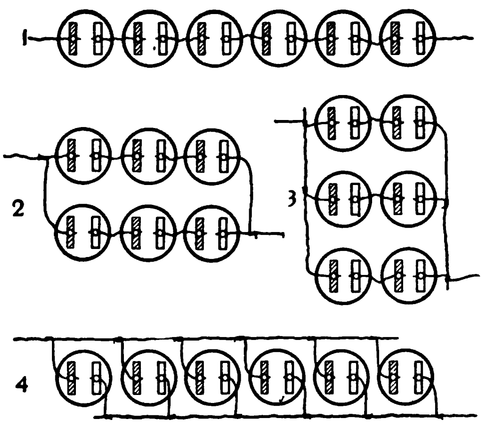


FIG. 111.

76. A dynamo giving a constant difference of potential of 55 volts is connected to a circuit wired for 20 lamps, any one of which may be turned on or off at pleasure. Each lamp has a resistance, when hot, of 50 ohms. An ammeter is connected in the circuit. What current will the ammeter record when one lamp is turned on? 10 lamps?

77. A dynamo intended to be used with 55 volt lamps is found to give but 48 volts when run at 1,600 revolutions per minute. How many revolutions should it make to give 55 volts?

78. Beneath a large shallow flat dish (see Fig. 112) place a sheet of cross-section paper having a number in each square.

Put enough acidulated water in the dish to fill it to a depth of about 1 cm., and dip the wires from a battery into the liquid, one near each end of the dish. Now place one wire from a galvanometer at any point, *a*, near one side of the dish and find a point, *b*, on the other side, such that when the other wire from the galvanometer is dipped in the liquid at the point no current will flow. Points *a* and *b* are at the same potential. Keeping *a* at the same place find three or four other points

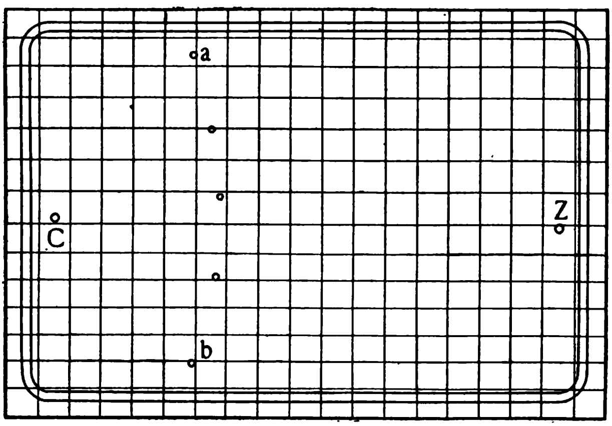


FIG. 112.

between *a* and *b* which are also at the same potential. Indicate all these points on a sheet exactly like the one under the dish and connect them by a line. Plot a number of such equipotential lines and then draw lines from *Z* to *C*, crossing the equipotential lines at right angles. These latter are lines of force.

79. The fall of potential in any part of a circuit is proportional to the resistance of that part of the circuit. The difference of potential at the terminals of a dynamo is found by the voltmeter to be 110 volts. The line is of copper wire, No.

12, which has a resistance of 0.005 ohm per metre. At the end of the line 1 km. from the station, what is the difference of potential between the wires? If you have lamps designed for 110, 105, and 100 volts which should you use at the end of the line? Which at a point halfway? Would it make a difference whether 2, 10, or 20 lamps were used? Note that if the potential is too high for the lamps they will be quickly "burned out," while if too low they will not be bright, but red.

80. (*a*) Connect a long wire to a gravity cell and determine the direction of the current in the wire by means of a compass needle. (*b*) Determine in the same way the direction of a current from a storage cell or two cells of plunge battery, then cut the wire, bare the ends, and dip them into acidulated water. The positive wire should be coated with copper oxide formed by the union of the oxygen, which collects there, with the copper.

81. A current of 1 ampère deposits 0.003277 gram of copper per second from a solution of copper sulphate. What is the strength of a current which deposited 1.26 grams of copper in one hour?

82. With a current of 10 ampères how long would it take to deposit a layer of copper .2 mm. thick over an electrotype having a surface of 20 sq. cm.?

83. Measure the length and diameter of a long piece of iron wire such as is used for supporting stove pipes and compute its resistance, taking the specific resistance of iron wire at 20° C. as 0.000014. Connect a battery through the iron wire and an ammeter and compute the resistance of the battery and ammeter combined. The resistance of the ammeter is usually so small as to be negligible. Cut the wire in halves and try the experiment with each half separately. Compare the three values obtained for the resistance of the battery. It is not to be expected that they will agree very closely, especially if the temperature of the room changes much.

84. Connect a galvanometer to a Daniell cell and hold a bar magnet in different positions above the needle to find what positions will increase the deflection and what positions will diminish it. It is usually possible so to adjust the field that the needle may be made to deflect between 30° and 60°, which is the best amount for all purposes. It is useless to try to measure the current by the deflection of a galvanometer when the deflection is nearly 90° or when the deflection is exceedingly small.

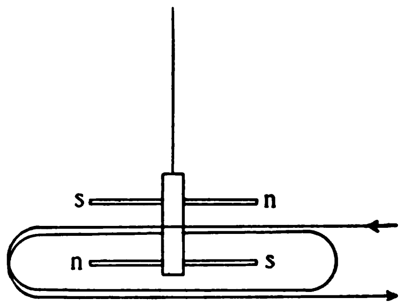


FIG. 113.

85. Suppose two small needles fastened together parallel on one support with their north poles opposite, as shown in Fig. 113. Would they be more or less sensitive to the effects of a current through the coil of wire than one of them alone would be? Such an arrangement is called an astatic pair. What does astatic mean?

86. Why should the coils of a galvanometer always be placed north and south when the galvanometer is to be used?

87. Connect by wires two metal handles to the terminals of the secondary of a small induction coil and test the physiological effects of the current. Connect the same handles to points on opposite sides of the break of a vibrating electric bell in such a way as to include the magnet in the circuit and test the bell for physiological effects. Has the bell a secondary circuit? Whence the effect? Test in the same way the strongest battery in the laboratory.

88. A common telephone consists of a bar magnet in front of which is supported, in the rubber enclosing case, a sheet of

soft iron. A coil of wire surrounds the magnet near the iron disk. When the coils of two such telephones are connected in series by wires as in Fig. 114, words spoken at T may be heard at T', a mile or more away.

(*a*) Explain how the motion of the iron diaphragm at T pro-

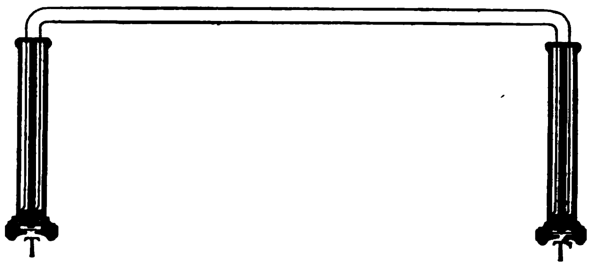


FIG. 114.

duced by the vibrations of the air causes induced currents in the coil, which, when conducted along the wire to T', set the diaphragm of that telephone vibrating. (*b*) Which telephone acts like a dynamo and which like a motor? Explain.

CHAPTER V.

WORK. ENERGY. MACHINES.

103. Work. Energy. — In our study of motions hitherto we have seen that motion of any sort could produce motions of other sorts, but we have not given much attention to the quantity of the effect which any given cause produces. We have studied the *how* of the phenomena rather than the *how much.* We propose now to review what we have learned and see if the separate topics cannot all be brought into closer relations than we have yet perceived to subsist between them.

We shall use the words *work* and *energy* in a somewhat technical sense and we must bear in mind that they are so used. We must be especially careful to note the distinction between *force* and *work.* A force is measured by the velocity which it imparts to unit mass in unit time. Unit force, when no other force is acting, gives unit velocity to unit mass in unit time. A dyne of force gives to one gram a velocity of one centimetre per second in a second of time. If this unit force moved the unit mass through unit distance it did unit work upon it. Work is measured by the product of the force acting times the distance through which it moved the body.

$$\textit{Work} = \textit{force} \times \textit{distance}$$

$$(16) \quad w = fl \qquad l = \frac{w}{f} \qquad f = \frac{w}{l}$$

If two forces balance each other so that no motion results no work is done.

Any units of force and distance may be used for measuring work. The *foot-pound* is the work done in moving a pound one foot against gravity. The *erg* * is the work done when the force of a dyne produces motion through a distance of 1 cm. The work done in lifting a gram 1 cm. is 980 ~~dynes.~~ ergs The foot-pound is equal to $980 \times 453.6 \times 30.48 = 13{,}548{,}703$ ergs. The erg is well adapted to the measurement of small amounts of work, the foot-pound, kilogram-metre, and so forth, for measuring large amounts.

Any body which has motion imparted to it gains thereby the ability to put other bodies in motion, or, in other words, the ability to do work. A ball fired from a gun in empty space would do no work until it should strike some body which offered resistance to its motion. When it had done an amount of work equal to that which was imparted to it, it would come to rest. Our earth is rushing through space at the rate of 18.5 miles per second. If it should strike another planet it would do work on a magnificent scale, but it meets with no perceptible resistance, and therefore does no work. It takes work to lift the hammer of a pile-driver high in air. So long as the hammer hangs there

* *Erg* from Greek *ergon*, work. The same root is found in en-erg-y, from Greek *energia*.

(see Fig. 115) it possesses the ability to do work — loose it and let it fall, it does work. The ability to do work we call *energy*. The energy of a body is the measure of the work which has been done upon it, less what it has since done, or it is the measure of the work the body must do before it can come to rest, if it is in

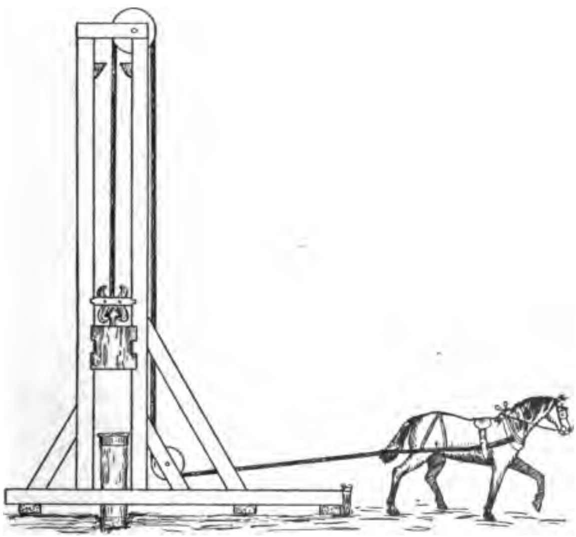


FIG. 115.

motion, or the work it must do to reach a condition of perfectly stable equilibrium, if it is at rest. The moving bullet and the lifted hammer both possess energy. Though they are not at the instant doing work, it took work to put them in their present relations to other objects. The energy of the bullet is energy of motion,

usually called *kinetic* * energy; the energy of the lifted hammer is energy of position or *potential* energy. Kinetic energy always implies that the body has been acted upon by a force, but if its quantity of kinetic energy is not changing there are no forces now acting. Potential energy implies that the body is in equilibrium under a balance of forces, the cessation of any one of which would result in motion. A bent bow is held in place by two hands pulling apart, the hand holding the string lets go and the arrow speeds away with energy, now kinetic, which was a moment ago potential in the strained bow and string. The sum of the energy of the system of bodies (bow and arrow) is unchanged. We say the energy has been transformed in kind, but unchanged in amount.

104. Conservation of Energy. — No physical change can occur without a transfer of energy, and conversely, every transfer of energy involves some sort of change in the motions of at least two bodies. We shall call all the bodies concerned in any particular transfer of energy a system. In all experiments where careful measurements are possible, it is found that similar measured quantities of energy will always produce equal effects of any particular kind. A moving ball which has a certain mass and volocity will generate a perfectly definite amount of heat when it strikes. A certain strength of electric current will produce in a

* Greek, *kineo*, move.

given time a perfectly definite amount of heat, or chemical action, or mechanical motion. The amount of chemical action, or heat, or mechanical motion, which was produced by the current mentioned, would, if spent entirely in producing current, produce exactly the quantity of electrical energy with which we started.

The most serious practical difficulty met with in carrying out such experiments is the tendency of energy to distribute itself to neighboring bodies in the form of heat. All careful experiments, however, lead to the one conclusion, — that energy is never gained or lost in any transfer, or, in other words, *the quantity of energy of a system of bodies concerned in a transfer of energy is absolutely constant.* This law, which is the product of no one man's labor, but represents the sum of the results of thousands of students of natural phenomena, is the great fundamental law of physics. It is known as the *Law of Conservation of Energy.*

If w be the quantity of energy lost by any one of a system of bodies and w', w'', etc., be the quantities gained by the other bodies in the transfer, then:

$$(17) \quad w = w' + w'' + etc.$$

If e, e', e'', etc., represent the energy of each body of the system at any instant, then:

$$(18) \quad e + e' + e'' + etc. = constant$$

so long as the system is kept from contact with bodies outside the system. In practice such perfect isolation of a system of bodies is exceedingly difficult to secure.

The more nearly perfect the isolation, however, and the more careful the measurements, the more nearly the results will be found to agree with the law. It is evident that we require for a systematic study of energy a system of units which shall comprise a separate unit for each sort of energy and a system of equivalents by which energy measured in one form may be compared with that measured in other forms.

105. Units of Energy. Equivalents. — The potential energy of a stretched spiral spring, as a spring balance, is measured directly in ergs if we know what force was used in stretching the spring and how much its length was increased, or, more exactly, how much the distance between the points of application, *A*, *B* (Fig. 116), of the two opposing forces was changed under the influence of the force.

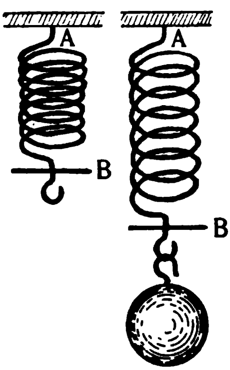


FIG. 116.

The potential energy of a ball which has been lifted to a height l is measured in ergs when we know the height l in centimetres and the force in dynes, which is 980 times the mass in grams. If the body should now fall towards the earth a distance of l centimetres it would possess an amount of kinetic energy equal to the potential energy which it has lost in falling the distance l. The body has a

definite velocity at any instant, as every moving body has. Let us try to find an expression for the energy in terms of mass and velocity. The velocity of a body which has fallen for t seconds under a constant acceleration, a, is

$$(2) \quad v = at$$

If it started from a position of rest its initial velocity was zero. Its average velocity, v', is therefore,

$$(19) \quad v' = \frac{0 + at}{2} = \tfrac{1}{2}\, at$$

But the distance, l, traversed in time t is

$$(1) \quad l = v't = \tfrac{1}{2}\, at \cdot t = \tfrac{1}{2}\, at^2$$

The force acting upon the body was

$$(4) \quad f = ma$$

and the work done in lifting it was

$$(16) \quad w = fl$$

which represents the energy it has gained in falling.

$$w = fl = mal = \frac{ma^2t^2}{2}$$

Since $at = v$, $a^2t^2 = v^2$, and

$$(20) \quad w = fl = \tfrac{1}{2}\, mv^2$$

That is to say, the kinetic energy of a body of mass m, moving with a velocity v, is measured by half the product of its mass times the square of its velocity.

A problem which engaged the attention of several physicists during the first half of the century just

closing was the determination of the *mechanical equivalent of heat*, or, as we may now express it, the measurement of the quantity of energy represented by a definite quantity of heat. Dr. Joule, of England, made the determination by means of a calorimeter in which a known mass of water was heated by the motion of a paddle wheel actuated by falling weights (see Fig. 117). The rise in temperature times the mass of the water measured the heat. He found the work represented by one calorie of heat to be 41,590,000 ergs. The factor 4.159×10^7 is known as Joule's equivalent and is represented by J. The value of J from the most recent determinations is 4.19×10^7. We may express calories in ergs by multiplying by J.

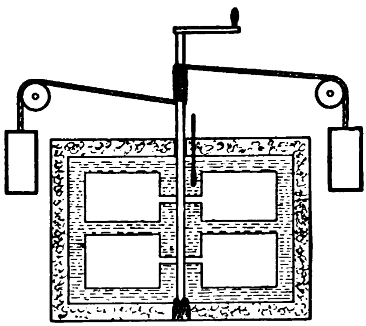


Fig. 117.

$$(21) \quad w = JH$$

The energy liberated or consumed during any chemical change may be measured by allowing the change to take place in a calorimeter and measuring the quantity of heat evolved or absorbed. In like manner, the energy of the electric current may be measured and compared with the energy supplied in the form of mechanical motion to the dynamo which generates the current.

Radiant energy, from the sun or any hot body, may be measured by allowing the radiations to fall upon a surface which absorbs the radiations and converts them into heat. Radiant energy consists of a wave motion which is transmitted through the ether. It becomes known to us only after it is converted into some other form of energy, as when in the form of light it affects our eyes (probably a chemical effect), or the sensitive photographic plate (certainly a chemical effect), or when it takes the form of heat, as all forms of energy are prone to do.

We shall discuss the nature of radiant energy more fully in a later chapter. For our present purposes it is enough to know that it is a form of energy which is transmitted at enormous velocities through what we call empty space as well as through matter, and that it may be transformed directly into heat or chemical energy and thus indirectly into other forms.

To recapitulate, we are accustomed to distinguish five forms of energy: (*a*) Mechanical energy, which is kinetic in a moving body, potential in a body which has been moved out of a position of stable to a position of unstable equilibrium; (*b*) thermal energy, which is kinetic when it raises the temperature of bodies, potential when it changes the state from solid to liquid or liquid to gaseous; (*c*) chemical energy, which is kinetic in the form of heat or the electric current at the instant the chemical change takes place, but is potential in all compounds which are not, like

the oxides, in the most stable state possible; (*d*) electrical energy, which is potential in the form of the static charge and in the magnet, but kinetic in the current; (*e*) radiant energy, which is always kinetic and is therefore the form which all energy tends to take, for this is the law of all change. Bodies which are in an unstable condition tend to lose the potential energy which they possess by virtue of their instability. For example, all masses which are lifted tend to fall; all springs which are bent tend to straighten; all gases tend to liquefy and all liquids tend to condense; all elements tend to oxidize or to undergo a change similar to oxidation; all electric charges tend to discharge. All of the changes mentioned generate heat which radiates off into space, and if the loss were not supplied by the radiant energy which comes to us from the sun our earth would soon become cold, motionless, and lifeless.

106. Energy and Life. — Why do living beings work, why must we work to live at all, and work much to live as we want to live? Our food, at once the material of which our bodies are built and the source of the energy of all our bodily movements, does not lie within our grasp. It must be sought, often pursued; when got it must be prepared. Our clothing, houses, implements, furniture, must be gathered in crude form and shaped to meet our uses, must be converted from its present condition to a less stable one, must have

work done upon it to fit it for our use. The earth which covers the stone in the quarry must be thrown aside, the stone must be broken in pieces, lifted out and carried to a new location, shaped with the chisel, set in place, before it is a part of the foundation of a house. Every one of these operations was resisted by a force, gravitation or cohesion, which opposed the change and necessitated the doing of work.

The man who shapes a tool of wood or iron must pull apart with knife or file the molecules, or bend and shape them into new forms. All these changes are opposed by forces to overcome which requires the doing of work.

As man's wants and desires increase he finds that more and more work is required if he is to satisfy them. The amount of energy which his own body can supply is limited. The civilized man can assimilate no more food than the savage. If he is to do the things he desires to do, he must find other sources of energy which he may utilize. They lie ready to his hand if only his brain can devise ways to use them.

107. Sources of Useful Energy. — Among the sources of energy available for man's use are (*a*) the bodily energy of animals; (*b*) the energy of flowing water and the tides and waves of the sea; (*c*) the energy of the winds; (*d*) chemical energy, particularly of combustion and of electric batteries.

Let us trace these secondary sources of energy back

as far as possible to see if any of them have a common source. The bodily energy of animals is derived from the food they eat, which consists of other animals and of plants. The animals eaten lived on plants, so that ultimately the source of food supply is almost wholly the plant kingdom. Now plants are able to use the stable compounds of the soil and air and convert them into unstable compounds useful for animal food only under the influence of sunlight. The sun is then the ultimate source of animal energy. The water which drives our mills does so because it is in a position to flow down hill. It possesses potential energy because it has been lifted to the clouds in the form of vapor by heat derived from solar radiation, and it delivers its energy to any one who can use it on its way back to the sea. The winds, too, are, as we saw when studying heat, convection currents in the air set up by heat received from solar radiation. The waves of the sea are due chiefly to the winds, hence windmills and wave mills are driven indirectly by solar radiation.

The energy of expanding steam which we utilize in the steam engine we derive from the chemical combination of the carbon of wood and coal with the oxygen of the air. Combustion is in fact a rapid oxidation of carbon or hydrogen. The coal was once living trees into whose composition the carbon was wrought after being torn from its stable union with oxygen by the energy supplied to the leaves of the plants by solar radiation.

Chemical batteries employ substances, like zinc and acid, which are not found in their present condition in nature but have had work done upon them by man. Batteries ought not to be reckoned therefore as primary sources of energy. The tides are due to the interaction of the earth and the moon, and do not therefore derive their energy from a distance. We see, however, that the sources of energy upon which we depend chiefly, and indeed almost wholly, are all traceable to one source — the sun. When we have used this energy or let it pass us by unused, as by far the greater part does and always will, it passes out again into that boundless ocean of ether whence it came.

108. Machines. — Man alone, of all animals, uses tools, implements, machines. In a broad sense a machine may be defined as a device by means of which energy is transformed so as to be advantageously applied. A man could dig a well (in some localities) with his hands alone, but give him a shovel with which he can pry the dirt apart and throw it out twenty handfuls at a time and he will work to better advantage than without the simple machine. If the well is deep he will gain an additional advantage if he use a rope and bucket to lift the clods to the top. He may even find it profitable to attach a windlass to his rope and use a much larger bucket than he could lift without it. A horse can carry a man or a small load on his back; hitched to a wagon on good roads he can draw ten

times the load. Sewing by hand goes slowly because the limit of speed is reached far sooner than the limit of strength. The sewing machine transforms strength into speed and multiplies the power of the seamstress sevenfold.

In our study of machines we shall perhaps proceed in the order of simplicity if we follow the historical order and take up first the traditional simple machines.

109. The Lever. — A rigid bar arranged to turn about a fixed point is called a *lever.* The fixed point is called the *fulcrum* (see Fig. 118). The two forces are applied at different points on the bar, P, P'. By the law of moments there will be equilibrium when the force f which acts at P times the distance PO is equal to f' times $P'O$, and the moments are opposite in direction. Both forces may act on the same side of the fulcrum, only so that their moments are opposite.

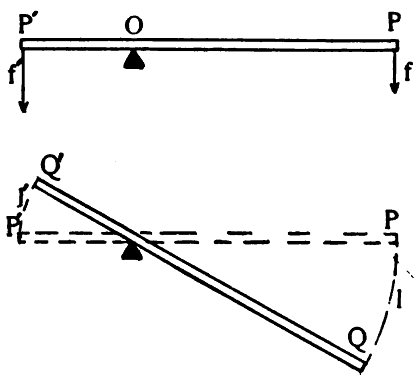


FIG. 118.

Let us now suppose the moment of the force f to be a little greater than f', so that

$$f \times \overline{PO} = f' \times P'O + \textit{the friction of the bearing}$$

There will then be motion in the direction PQ. Then the work done by the force f in moving the distance $PQ = l$ is fl and the work done at P' is $f'l'$. Since l and l' are arcs of circles whose radii are PO and $P'O$ respectively, it follows from geometry that

$$(a) \quad \frac{PO}{P'O} = \frac{l}{l'}$$

But from the law of moments

$$(b) \quad f \times PO = f' \times P'O \quad \text{or} \quad \frac{f}{f'} = \frac{P'O}{PO}$$

Whence it follows from (a) and (b) that

$$(22) \quad \frac{f}{f'} = \frac{l'}{l} \text{ and}$$

$$(23) \quad fl = f'l'$$

when the forces are in equilibrium. When motion is produced, $fl = f'l' +$ work done in overcoming friction, w'', whence

$$(17) \quad w = w' + w''$$

which is the law of work.

In the older treatises on mechanics the force applied, f, was called the *power* and the force obtained, f', was called the *weight*. The words *power* and *weight* have very definite meanings in physics and meanings not at all alike. It seems best not to retain these words, therefore, in our discussion of machines.

The distances PO and $P'O$ being straight lines are easier to measure than the arcs PQ and $P'Q$. Their ratio may be used therefore for the ratio of l to l', to which it is always equal.

The lever has numerous applications. The spade, the handspike or crowbar, the pump handle, all serve to enable a man to overcome forces greater than he is able to exert, but always at the expense of distance. In every case the hand moves a greater distance than the object at the other end of the lever.

The ratio $\frac{f'}{f}$ is called the *mechanical advantage* of the machine. It is to be borne constantly in mind that the real advantage consists in obtaining the energy in a form better suited to our needs. The traveller who gets four marks for a dollar in Germany or five francs for a dollar in France is pleased because he gets what he needs, and he is even willing to pay the broker a small percentage for making the exchange. In mechanical transfers we must always pay exchange too. It is in the form of heat and is always delivered on the spot. We make it as small as possible by avoiding friction and other sources of heat waste, but no rational mechanic expects to get back all he puts in, much less win any energy by means of a machine.

A machine which transfers energy with but little loss is said to be an efficient machine, and the ratio of w' to w is called the *efficiency* of the machine. Since $w = w' + w''$ and since w'' is never zero, it follows that w'/w is always less than unity, or, as usually expressed, less than 100 per cent.

$$\textit{Efficiency} = \frac{\textit{work obtained}}{\textit{work expended}}$$

$$(24)\quad E = \frac{w'}{w} = \frac{w - w''}{w}$$

where w'' represents the total work not obtained in useful form but lost — usually in overcoming resistance in the machine itself. The useful work obtained, w', is spent in overcoming resistance on objects other than the machine, and may take any of the well-known forms of kinetic or potential energy.

The efficiency of a lever depends very much on the amount of friction at the fulcrum, but very much more upon the direction of application of the force. When a lever rests upon a knife edge, as in the balance, friction for small loads may be made exceedingly small in amount, but when a similar arrangement is used with the crowbar there must be friction enough to keep the bar from slipping. The use of the crowbar is limited by the fact that as soon as the object has been lifted a short distance, it must be supported while the fulcrum is raised so that the operation may be repeated.

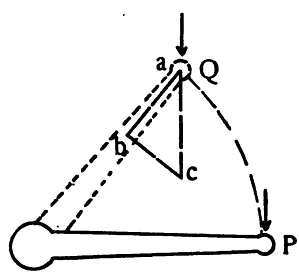


FIG. 119.

Where pressure is applied from a single direction while the lever is allowed to make a full revolution, as in the crank of a bicycle, there is only a small part of the revolution, near P (Fig. 119), where the whole of the force applied produces rotation. At Q the component bc producing rotation is smaller

than the component ab, which is wasted in producing pressure on the bearing. At P the efficiency is nearly 100 per cent, while at Q it is less than 50 per cent, and at the top and bottom the force applied is almost all wasted. We overcome this defect in a large measure by applying our force mainly during that part of the revolution where it will accomplish most.

110. The Pulley. — If the force were applied to a lever by means of a rope acting downward at the end of the lever we should encounter the same disadvantages as in the bicycle crank, but if instead of a lever we use a grooved wheel over which a rope passes, the force may always be applied at the proper point, while the fulcrum may be placed once for all higher than the point to which the load is to be lifted. If the wheel be thought of as having spokes (see Fig. 120) it is evident that the horizontal pair of spokes may be considered the two equal arms of a lever at the instant when the wheel is in the position shown. Since the wheel is practically solid any diameter may be such a lever. Friction keeps the rope in position on the wheel without greatly retarding the motion. The pulley is free from the defects of the lever, but it offers in its simple form no mechanical advantage whatever. Since $l = l'$, f must

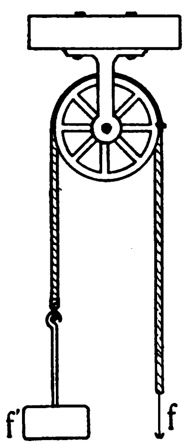


FIG. 120.

equal f' and more. The pulley even in this simple form is useful, however, since it changes the direction of the application of the force. A man can lift a large bucket of water from a well more easily by pulling downward on the rope than by leaning over the well and pulling upward, while a horse cannot conveniently pull in any direction but the horizontal. The two fixed pulleys used in the pile driver (Fig. 115) enable us to employ a horse for lifting the weight. By the arrangement of pulleys shown in Fig. 121 the load which is attached to a movable pulley moves but half as far as the point of application of the force moves, and we have a mechanical advantage of 2, while the use of a third pulley as in Fig. 122 gives us a mechanical advantage of 3.

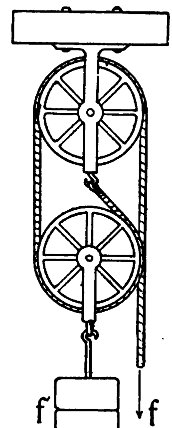


FIG. 121.

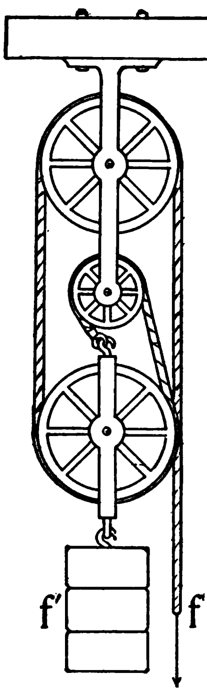


FIG. 122.

111. The Wheel and Axle. — The winch or windlass, the capstan, the wheel and axle combine the principles of the lever and the pulley. They all consist of

an axle about which a rope is wound drawing the load. The load is moved by force applied at the circumference of a larger wheel or at the end of a crank. Fig. 123 shows a common winch or windlass. In machine shops combinations of pulleys of various sizes on the same shaft, connected by belts with pulleys on various other machines in use, make it possible to run any machine in a shop at any desired speed from an engine whose speed is constant. When it is desirable to run the same machine at different speeds, as is the case with the lathe, a pair of cone pulleys is used. See *C*, *C′* (Fig. 124), where *D* is the driving pulley of the engine, *SS′* a line of shafting from which attachments are made either directly

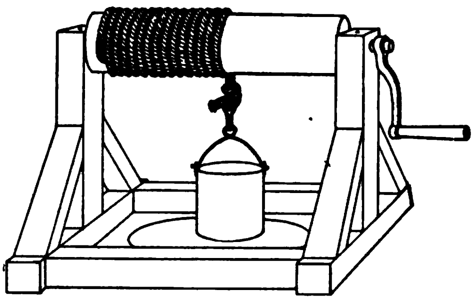


FIG. 123.

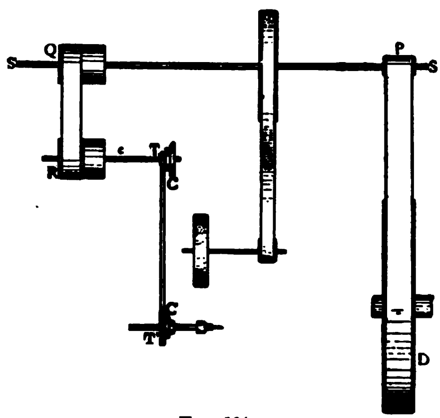


FIG. 124.

to other machines or to countershafts from which the energy is conveyed to the machines as wanted. With the belt on the cone pulleys as shown, the lathe has its lowest speed. The cones are of such dimensions that the same length of belt will fit in all cases.

When two shafts are connected by pulley and belt the one carrying the larger pulley must go the more slowly, since all parts of the belt travel at the same speed, and a point in the circumference of either pulley travels with the speed of the belt. In Fig. 124, for example, the driving wheel, *D*, has a speed of 2,000 revolutions per minute, and a diameter of 120 inches; *P* has a diameter of 12, *Q* of 24, *R* of 24, *T* of 5, and *T′* of 20. To find the speed of *C′* we have only to multiply 2,000, the speed of the driving pulley *D*, by the ratios of the sizes of the successive pairs of pulleys, thus:

$$\textit{Speed of } C' = 2{,}000 \times \frac{120}{12} \times \frac{24}{24} \times \frac{5}{20} = 5{,}000$$

Where great compactness is desired the two wheels are brought into direct contact by cogs. A series of such cogwheels is called a train of wheels. By a proper combination of sizes the train of wheels in a clock is made to drive the hour, minute, and second hands at the proper relative speeds. In cases like the bicycle, where a belt would slip and the wheels cannot conveniently be placed in contact, sprocket wheels and chain accomplish the purpose. The rear wheel may be given any desired speed by suitably choosing the relative size of the two sprocket wheels.

112. The Inclined Plane. — When a body is resting on an inclined plane its weight is supported partly by the plane, partly by some force acting parallel to the plane. The ratio of the two forces is the ratio of the two components, bc, ac (Fig. 125). The ratio of the force overcome to the force applied is

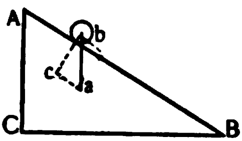


Fig. 125.

$$(25) \quad \frac{f'}{f} = \frac{ba}{ac}$$

But since the angles B and b have their sides mutually perpendicular, triangles ABC and abc are similar and

$$\frac{ba}{ac} = \frac{BA}{AC}$$

Whence it follows that

$$(26) \quad \frac{f'}{f} = \frac{BA}{AC}$$

that is to say, the advantage of an inclined plane is the ratio of the length to the height of the plane, provided that the force acts along the plane.

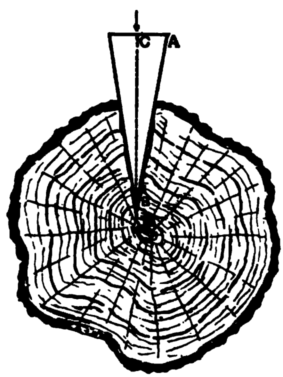


Fig. 126.

The *wedge* may be treated as a pair of inclined planes placed base to base (see Fig. 126). The force is applied in the direction CB to overcome the force of cohesion which is in a direction at right angles to CB. It follows that

$$(27) \quad \frac{f'}{f} = \frac{CB}{2\,AC}$$

When wedges are used to split large stones a row of holes is drilled along the line on which the stone is to be divided and several wedges are driven at the same time. If the only object aimed at is to shatter the stone a charge of powder or dynamite is often used.

113. The Screw. — When a wagon road or a railway is to be carried up a very steep mountain it does not go straight up, but winds in a spiral, making the ascent by going several times the distance. The length of the incline is thus increased without increasing its height, thus

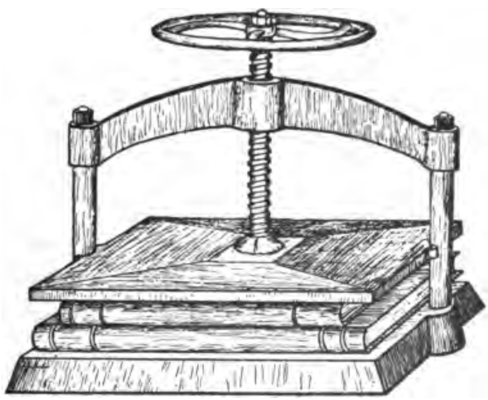


FIG. 127.

greatly increasing the mechanical advantage. The same principle is employed in the *screw*, where the advantage is usually still further increased by the use of a wheel or a lever at the head of the screw (see Fig. 127). The distance which the screw advances in one revolution is the distance, d, between the adjoining threads of the screw measured parallel to the axis of the screw, while the distance moved by the applied force is the circum-

ference, c, described by the point of application of the force. The advantage of the screw is therefore

$$(28)\quad \frac{f'}{f} = \frac{c}{d}$$

Thus a force of 2 pounds applied to the handle of a screw-driver 2.1 inches in diameter (= 6.6 inches in circumference) would exert upon a screw having 12 threads to the inch a force of 158.4 pounds.

The simple machines just described — the lever, the wheel, the incline (including the screw) — have been known and used for thousands of years. The innumerable mechanical contrivances in use to-day, from the wheelbarrow to the typesetting machine, are all but combinations of the simple machines. It is when we seek to employ other forms of energy than those known to the ancients (animals, wind, waterfalls) that we meet machines involving principles not found in the simple machines, but always associated in practice with them.

114. Heat Engines. — As long ago as B.C. 200, machines employing the expansive force of heated air and steam were described by Hero, or Heron, of Alexandria. He arranged the device shown in Fig. 128, by which a fire kindled on the hollow altar causes the air within to expand, forcing the water out through the siphon into a bucket, where its weight served to open the temple doors. When the fire was allowed to die out the doors closed as miraculously as they had opened.

Hero's steam engine is shown in Fig. 129. It is easily constructed. Steam from the basin enters by one of the hollow columns supporting the ball, which is made to revolve by the reaction of the air upon the escaping steam.

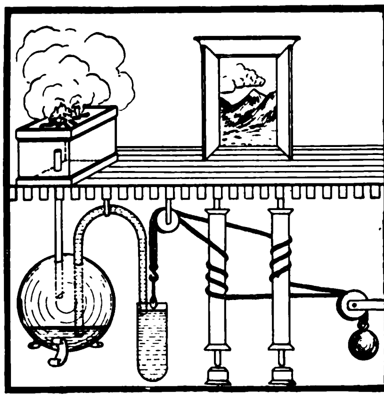


FIG. 128.

The steam engine invented by Watt in 1765 was the outgrowth of various forms of the "atmospheric" engines which were the fruit of Guericke's invention of the air pump a hundred years before. In these engines a vacuum was produced in a cylinder by the condensation of steam within it, and a piston was pushed into the cylinder by the pressure of the atmosphere. Such engines were used for pumping water from mines, but were very wasteful of steam.


FIG. 129.

Watt not only invented the steam engine, which with slight modifications is used

to-day, but he also invented most of the devices by which it is controlled and rendered efficient, such as the automatic governor and throttle valve. The slide valve was invented by his assistant, Murdoch.

The principle of the steam engine may be understood from Fig. 130. The steam from the boiler enters the steam chest at *S*, whence it is admitted to the cylinder through the ports, *P*, *P′*, by means of the

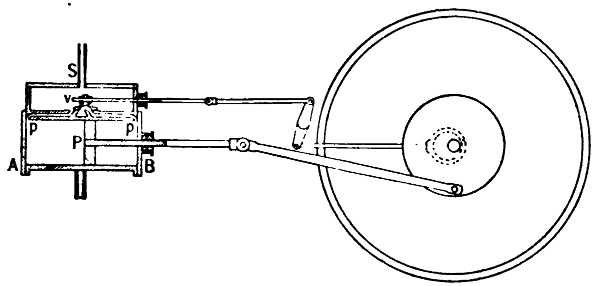


FIG. 130.

slide valve, *V*, which opens port *P* while the piston is near the end *A*, at the same time closing the passage from *P′* to the steam chest and opening it to the exhaust, thus allowing the condensed steam to escape at atmospheric pressure. By the time the shaft has made half a revolution the piston is in suitable position at the other end of the cylinder, the valve has moved so as to open *P′*, and live steam is now admitted to the right-hand side of the piston head, while condensed steam escapes through *P* to the air.

The oscillating motion of the piston is converted into rotary motion by a crank attached to a shaft which carries a large driving pulley. Attached to the same shaft is a smaller crank or eccentric which operates the slide valve.

In the best steam engines not more than twenty per cent of the energy liberated in the combustion of coal is utilized, yet coal is so cheap that it continues to furnish most of the energy used for manufacturing purposes.

115. A form of engine which is coming quite generally into use where but a small amount of power is required is the so-called gas engine (Fig. 131). The propulsive force is furnished by the explosion produced by igniting a mixture of air and coal gas or gasoline vapor within the cylinder. The gas mixture is admitted only at every other revolution, since the products of combustion must first be expelled by the piston on its first return stroke. During the second stroke the mixed gases are admitted through a valve, which closes like a pump valve when the piston starts back. When the piston is at the end of its stroke and has compressed the gases it closes an electric circuit,

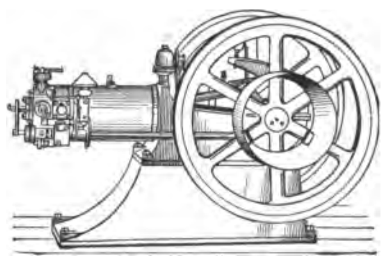


FIG. 131.

which is broken when the piston starts on its second outward stroke, producing a spark which ignites the gases, and the cycle of operations is then repeated.

The fact that the force is exerted on but one side of the piston and but once in two revolutions makes the engine less steady than the steam engine, with its two impulses for each revolution. This fault is overcome to some extent by the use of heavy fly wheels.

Regulation is accomplished by a governor, which entirely closes the supply valve when the speed exceeds a certain amount. When the load is light the explosions occur only at rare intervals, so that but little gas is wasted. Gas engines are now used to drive self-propelling carriages, a fact which is likely to lead to a very light and compact form of machine. At the present time it is more expensive than the horse as regards first cost, if not as regards maintenance.

The gas engine has a higher efficiency than the steam engine, yet it has not nearly reached the highest efficiency of which it is theoretically capable.

116. Rate of Doing Work. Power. Activity. — The amount of energy which any machine transforms in unit time is its rate of doing work, or, briefly, its *power* or *activity*. Watt introduced the term *horse-power* as the unit of activity, and defined it as 33,000 foot pounds per minute. The metric unit is the *watt.* It is defined as 10,000,000 ergs per second $=$ 1 joule per second $=$ 1 volt $\times$ 1 ampère.

$$(29) \quad 1\ h.p. = 746\ watts$$

The horse-power of a dynamo is usually expressed in kilowatts. Thus a 50 k.w. dynamo would furnish 50 ampères at 1,000 volts and would require a 67 h.p. engine to run it. If we now recall that a calorie is 4.19 joules, we see how easy it is to measure both energy and power in any one of its forms, and then to compare the quantity measured with the same quantity measured in another form. This enables us to find the efficiency of a machine like the heat engine, which transforms energy from one form to another.

117. Other Engines. — The applications of the laws of energy and the principles of machines to water wheels, tide wheels, and windmills may be left as an exercise for the ingenuity of the student as he may have opportunity to study them. He has only to remember that no exception has ever been found to the law of the conservation of energy, and he may feel very sure that the man who claims to have invented a machine which will run and do work, even enough to overcome friction, after the energy supplied to it has been expended, is either deceived himself or is trying to deceive others for his own profit. The man who is pursuing the phantoms of perpetual motion should devote himself to solving some of the many problems which offer sure reward to the successful inventor.

No man can hope to create energy. No man need wish to do so, for energy by the millions of horse-power

is going to waste constantly at our very doors. The energy contained in the water which falls on the roof of a large hotel in a year would, if utilized, run the elevator twelve hours a day the year round. The energy of the sun which falls on a large factory and is radiated again into space would, if all utilized, run all the machines in the factory. A small fraction of the energy of the wind which blows across every farm would, if harnessed, do all the work of the farm.

The men who are to profit themselves and their generation by their inventions must first master the known laws of nature and then apply them to existing problems.

118. The Storage of Energy. — If the energy from so intermittent a source as the wind, for example, is to be utilized to any large extent by people who must work every day, means must be found to store the excess of one day against the needs of another day. On the other hand, some forms of energy, like light, are consumed principally at certain hours of the day, and the engine must have power enough to supply the greatest demand, though an engine a third as large could do the work if it were distributed uniformly throughout the day. Many large lighting stations now have large storage batteries which are charged during the middle of the day and help carry the load in the evening. Indeed, the storage battery is as yet the only successful device for storing energy on a large scale.

Plants, as we know, store the energy of the sun's rays, but the supply thus stored could not meet our demands if the treasures stored in the coal fields during the ages rich in carbon were exhausted. Ere the coal fields are all used ways will perhaps be found to collect and store the solar energy by artificial means which will make us independent of coal, as iron and paper are making us independent of wood, and as aluminum might easily make us independent of copper and tin.

119. The Transference of Energy from Place to Place. — It often happens that large waterfalls are located at some distance from a mine, where it would be very desirable to use the energy of the falling water. The desired end may be accomplished by converting the energy into the form of an electric current, when it may easily be carried by wires to a considerable distance, to be again converted by means of motors into mechanical motion.

In large towns there are usually a number of small shops, printing offices, etc., which need a small amount of power at somewhat irregular intervals. The total amount of power needed can be supplied from one large engine, with one or two men to care for it, much more economically than by a number of small engines.

Several plans have been used for distributing the power. The pressure applied to water by a large pump at a central station may be used by water motors

at any place on the line of pipes. The friction of moving water on the pipes is, however, so great that very large pipes are required both for the inflow and the escape of the water, so that the ordinary service pipes are not sufficient to furnish any considerable power at the pressures usually employed. Compressed air furnishes a simple means of conveying power, but one which has not come into general use because of the expense of laying pipes of sufficient strength and size.

Here again the electric current is found to be admirably adapted to the distribution of power. Electric conductors may be carried anywhere and tapped at any point; indeed, the point of contact may be continuously changing, as in the trolley car, without any serious loss by leakage. The ease with which the current may be turned on or off or varied in amount at pleasure, and the variety of uses to which it may be put — heat, light, mechanical work, electroplating — make the electric current the ideal form of energy for transference to a distance and for distribution in small quantities.

When the energy is to be carried long distances high potentials are used, because the heat lost in the wire is proportional to the square of the current strength. Thus if 100 kilowatts are to be carried at 100 volts, we have

$$100 \text{ k.w.} = 100 \text{ volts} \times 100 \text{ ampères}$$

but at 1,000 volts we have

$$100 \text{ k.w.} = 1{,}000 \text{ volts} \times 10 \text{ ampères}$$

In the first case the current strength is ten times that in the second, and the heat lost in the wire is 10^2 or 100 times as much for a wire of a given size.

Exercises.

89. (*a*) Two men lift a 200 pound barrel of salt upon a wagon 3 feet high. How much work do they do? (*b*) One man rolls the same barrel of salt upon the same wagon, using an inclined plane 9 feet long. How much force does he exert? How much work does he do? (See Fig. 132.).

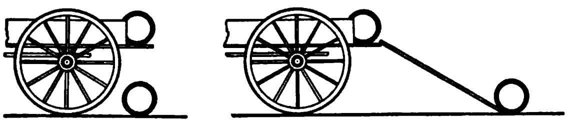


FIG. 132.

90. A man weighing 150 pounds carries 50 pounds of brick up a ladder 20 feet high. (*a*) How much work has he done? (*b*) How much useful work? (*c*) What is the efficiency of this method of working? If he can draw up 100 pounds to the same height and in the same time by means of a single pulley and a box weighing 20 pounds, friction being equivalent to 30 pounds, (*d*) how much work does he do? (*e*) How much useful work? (*f*) What is the efficiency of the method used?

91. A horse which can exert continuously a force of 500 pounds is used to lift the material for a building. A number of iron girders, each weighing 600 pounds, are to be lifted. What combination of pulleys will just use the full strength of the horse if 20 per cent of the work he does is spent in overcoming friction?

92. A certain class of freight engines can pull a load of 100 tons up a grade of 2 feet in 100. Suppose all grades on the line reduced to 1 foot in 100, what load can these engines pull

if we suppose the work done in overcoming friction equal to ¼ the work done by the engines?

93. A crane (Fig. 133) employs a system of 5 fixed and 5 movable pulleys and a windlass having an axle 8 inches in diameter, to which is attached a wheel having 40 cogs driven by a wheel having 8 cogs, to which is attached a crank 16 inches long. How large a stone can a man lift if he applies a force of 25 pounds to the end of the crank? No allowance is to be made for friction.

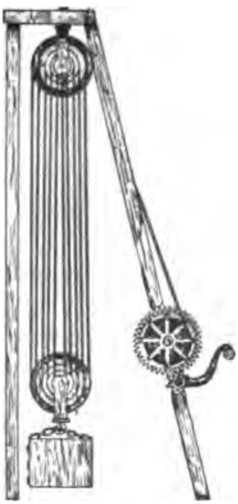


FIG. 133.

94. (*a*) How much work is required to lift a ton (2,000 pounds) of coal from a mine 500 feet deep? (*b*) What must be the power of an engine to lift 20 tons per day from this mine?

95. How much coal would the engine in the last exercise consume per day if it converts 15 per cent of the energy of combustion of coal into mechanical work? The heat of combustion of coal is 7,800 calories per gram.

96. A man lifts his own weight, say 150 pounds, 2 inches every step he takes. What is his rate of working if he takes steps 2½ feet in length and walks on a level at the rate of 3 miles per hour?

97. A man weighing 145 pounds propels himself at the rate of 10 miles an hour on a bicycle weighing 25 pounds over level pavement. If he should remove his feet from the pedals at any instant, the wheel would run 528 feet before being brought to rest by friction. What is the man's rate of working?

98. A storage battery is charged at the rate of 20 ampères

for 12 hours. It will give a current of 10 ampères for 20 hours. What is its efficiency?

99. The battery mentioned in the preceding exercise has a difference of potential of 50 volts. (*a*) When discharging at 16 ampères, what is its power in watts? (*b*) If a motor when connected to this battery consumes 8 ampères at 60 volts and does work at the rate of $\frac{1}{2}$ horse-power, what is the efficiency of the motor?

100. A freight train weighing 100 tons is moving at the rate of 20 miles an hour. (*a*) How much kinetic energy has it? (*b*) How much energy would there be lost if the train were brought to rest by applying the brakes? (*c*) Suppose that a dynamo were connected to the axle of one of the cars and made to store energy in a battery, or that a pump similarly attached were used to compress air in a reservoir, could a part of this waste energy be utilized?

101. Explain why more coal must be burnt to run a local train a given distance than to run a through train of the same size the same distance.

CHAPTER VI.

VIBRATIONS. WAVES.

120. Vibratory Motion. — Most bodies which have not been recently disturbed are at rest in that position in which they have the least possible potential energy: a ball hung from the end of a string hangs with the string in a vertical position, the ball being as near the earth as it can get; a vessel of water has its surface horizontal; a dew-drop is spherical. If we draw the ball to one side (*P*, Fig. 134) and let it go, the potential energy we have thus imparted to it will be converted into motion, but the ball will not stop in its former position of rest, since its momentum will carry it past *O* to *P′*, a point almost as far from *O* as *P* is. After coming to rest at *P′* it will return through *O* almost to *P*, repeating the operation until the work done by it in pushing aside the air has consumed the energy given it when it was pulled aside.

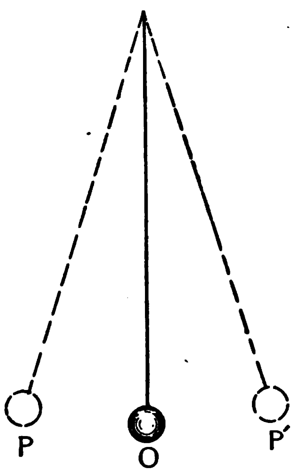


FIG. 134.

If a vessel partly filled with water be tilted as in Fig. 135, and then quickly restored to its original position, the water at O will alternately rise and fall between P and P' and finally come to rest at O.

Such motion is called *vibratory* motion. The distance PO is called the *amplitude* of vibration. The time occupied by the particle in going from P through

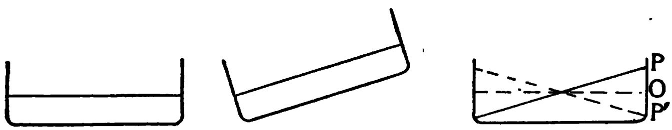


FIG. 135.

O to P' and back again to P is called its *period* of vibration, and the number of vibrations made in one second is called the *rate* of vibration. All vibratory bodies which have a definite and constant period of vibration are said to have *harmonic motion.* It is called harmonic motion because all musical sounds are due to motion of that sort. Such motion is indeed very common because most bodies are in a state of comparative rest in stable equilibrium, but are frequently disturbed by small forces which are able to set them in vibration, though not great enough to move them from their places. Thus the boughs of a tree sway to and fro in every breeze, yet stay in one place for scores of years. Whenever the force of displacement, as likewise the force of restitution, increases directly with the displacement, the motion will be harmonic and the period of

vibration constant. We can easily prove that the force required to stretch a spiral spring is proportional to the elongation produced in the spring, for the scale of a spring balance is a scale of equal parts. If we hang a small weight from a light spiral spring, pull it downward and release it, it will vibrate with a slowly diminishing amplitude, but with a constant period. The hairspring of a watch (Fig. 136) vibrates in the same way. Vibrating pendulums and vibrating springs furnish us, therefore, with the means of measuring time.

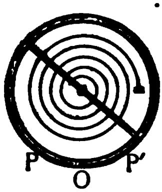


FIG. 136.

121. The Pendulum. — The pendulum furnishes a good example of harmonic motion. The force of restitution, that is, the force tending to make the ball return to its position of equilibrium, is gravity. It is obvious that when the ball (see Fig. 137) is anywhere except at O, gravity may be resolved into two components, one of which acts to restore the ball to its position of equilibrium. This component is greater at P than at Q. The force of restitution is greater the greater the amplitude. Since there is always a force acting toward O, the motion is accelerated toward O, that is to say, the ball will go faster and faster as it approaches O, but slower and slower as it leaves O. The energy of the pendulum at P and at P' is all potential, while at O it is all kinetic. At intermediate points it is partly potential and partly kinetic.

Galileo noticed that the great chandelier in the cathedral at Pisa seemed to swing in equal times. He placed his finger on his pulse and verified the truth of his first impression. Further experiment showed that the time of a pendulum depends upon its length, which must be measured from its centre of mass to the point of support, and with the force of gravity at the place. It may be proved mathematically that the period, T, of a pendulum of length l, is

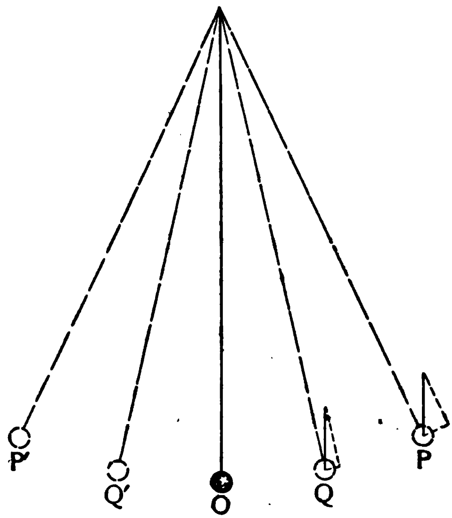


FIG. 187.

$$(30)\quad T = 2\pi\sqrt{\frac{l}{g}}$$

where $\pi = 3.1416$ and g is the force of gravity. The time of oscillation, t, which is the time between two passages of the ball through O, is just half as great:

$$(31)\quad t = 3.1416\sqrt{\frac{l}{g}}$$

A pendulum whose time of oscillation, t, is one

second is called a *seconds pendulum.* Its length may be easily found from the equation (31)

$$t = 3.1416\sqrt{\frac{l}{g}}$$

$$t^2 = (3.1416)^2\,\frac{l}{g}$$

$$(32)\quad l = \frac{t^2 g}{(3.1416)^2}$$

The same formula will give us the value of g when we have measured t and l.

$$(33)\quad g = l\left(\frac{3.1416}{t}\right)^2$$

It will be seen that the only variable quantities in this formula (31) are l and g. It follows that the mass, size, and material of the pendulum have no effect on its time of oscillation. It is to be remarked that formula (31) is true only on the supposition that the amplitude of vibration is small as compared to the length of the pendulum.

A *simple pendulum* is one in which the weight of the string is so small that the centre of mass of the pendulum can be thought of as identical with the centre of mass of the bob, which must be small. If the bob is a ball or disc we measure l from the centre of the bob to the point of support.

The length of a pendulum is changed by temperature, and various plans have been devised for correcting the error thus caused. Descriptions of compensating

pendulums may be found in the larger treatises on physics or in any good cyclopædia.

122. Wave Motion. — The particles of solid bodies are so related to each other that if any particle be set in vibration it will impart its motion to the particles nearest to it, these particles in turn passing the motion to their neighbors. Thus the slamming of a heavy door in a large building will send tremors to every part of the building. The explosion of a powder magazine will shake buildings miles away.

Fluids also transmit disturbances, not by being distorted as solids are, for they have no form, but by being compressed.

A disturbance which is transmitted from particle to particle through a medium is called a *wave*.

There are but two ways known to us in which energy may be transmitted from body to body: (1) by the motion of bodies themselves, as when a bullet is fired or a stream of water flows; (2) by waves, as when pressure is imparted to the water in closed pipes or to the ether surrounding the sun, and the pressure is transmitted to all points in any way connected with the source of energy.

Every vibrating body sends waves through the bodies it touches. Wave motion is therefore present everywhere. As heat and sound and light it assails us on every hand. The study of wave motion is, therefore, of the highest interest to us.

We have all noticed that a pebble dropped in still water will send a wave to every part of the pool. The reason is obvious: the water displaced by the pebble heaped upon the water near it presses upon it, falls, and is carried as far below the level surface as it was above it first, as the water did in the dish shown in Fig. 135.

The pressure is transmitted to all points of the water until *in time* the wave has reached every point.

When the water at any point as P (Fig. 138) has

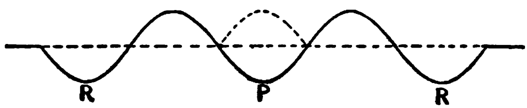


FIG. 138.

made a vibration and a fourth, and is starting up for the second time, there are a number of particles at equal distances from P which are just starting up for the first time (R, R). The distance PR is called *a wave length*, and all particles which are any whole number of wave lengths distant from P are said to be in the same *phase*. The distance PR we shall call l. It is evident that if every vibrating particle in the medium makes n vibrations per second, the rate of propagation of the disturbance, or, as we say, the velocity, v, of the wave, is

$$(33) \quad v = nl$$

This expression is true for all kinds of wave motion. The velocity of waves of any sort in a particular medium is constant, but when the waves pass into a different medium the velocity will, in general, change.

123. Direction of Waves. — A wave which travels along the bounding surface between two fluids, like air and water, travels outward in concentric circles. A wave in a string travels along the string. A wave in a large body of any homogeneous medium travels outward from the centre of disturbance in concentric spheres. The direction of vibration of the particle may be the same as the direction of propagation of the

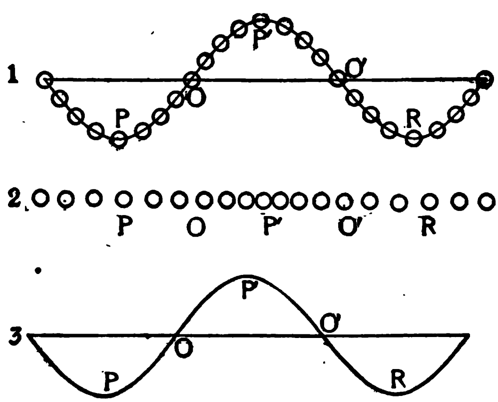


FIG. 139.

wave: the wave motion is then said to consist of *longitudinal* vibrations; or the particles may vibrate in a direction at right angles to the direction of propagation of the wave: the vibrations are then said to be *transverse*. The waves on the surface of water and the waves which are sent along a clothesline when it is struck with a stick are transverse. The waves of pres-

sure which are imparted to water in pipes by means of a force pump are longitudinal.

The two sorts of waves may be represented diagrammatically as in Fig. 139, where the circles represent particles of the medium in various phases of vibration. At 1 a transverse wave is shown, at 2 a longitudinal wave. To save time in drawing the figure it is customary to represent both sorts of waves as at 3, leaving the sort of wave to be stated in the discussion. The distance $PR = l$ is a wave length, and P and R are in the same phase, while P and P' or O and O' are in opposite phases.

124. Reflection. — It was remarked above that waves change their velocity on entering a different medium. It is to be observed further that the energy of a train of waves in any medium is never all transmitted to the second medium; a part of it is always reflected or turned back at the bounding surface. If the velocity of the waves is less in the second medium than in the first, the reflected wave will be opposite in phase to the original wave (see Fig. 140). If the velocity is greater in the second medium the wave will be reflected without change of phase (see Fig. 141).

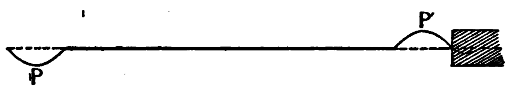


FIG. 140.

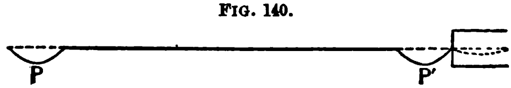


FIG. 141.

Except in the cases of large bodies of water and the atmosphere we have to deal with waves in a body of limited size, and must therefore have to do with waves at bounding surfaces. It will be well for us, therefore, to examine the subject somewhat more in detail.

125. Addition of Waves. Stationary Waves. — When two waves pass any portion of a medium at the same time, each wave produces the same effect on every particle of the medium as if it alone were passing. This is in accordance with Newton's second law of motion. The resultant motion is, therefore, the geometrical sum of the two motions, and the resultant wave may be quite different from either of the two components. If one end of a rubber tube be attached to a hook in the ceiling while the other end is fastened to a hook in the floor or table, waves may be sent up the tube by striking a quick blow near the lower end of the tube. When a wave reaches the top it will be reflected with a change of phase. If at the instant the reflected wave starts downward a second one is sent up from S (Fig. 142), the two waves will meet midway and interfere. It is easy to see that a particle halfway up (Fig. 143) is pulled in opposite directions by almost equal forces and will therefore remain at rest. If a continuous series of waves be sent from S at equal intervals they will all interfere

FIG. 142.


FIG. 143.

so as to produce rest at the middle but motion in the other parts of the tube, which will appear as in Fig. 144. The point at rest is called a ***node***, while the points of greatest activity are called *antinodes*. We shall indicate nodes by N, antinodes by A hereafter. By timing the waves suitably we may produce two nodes besides those at the end, as in Fig. 145, or three or four or more at pleasure. In every case the distance between two nodes is half a wave length.

FIG. 144.

FIG. 145.

$$(34)\qquad \overline{NN} = \overline{AA} = \frac{l}{2}$$

The case just cited is an illustration of the general case shown in Fig. 140, where $v_1 > v_2$. The case where $v_1 < v_2$ may be illustrated if we fasten the upper end of our tube to a string which is fastened to the hook as shown in Fig. 146. The reflected and incident waves (as waves sent from S are called) will evidently interfere in such a way as to reënforce each other as they pass; the middle portion will not be a node, neither will the upper end, but the nodes will be situated as shown in Figs. 147 and 148.

FIG. 146.

FIG. 147.

FIG. 148.

126. Law of Reflection. — The wave in a rope or

rubber tube is reflected back along the tube as a matter of course, but when a water wave, for example, strikes a pier the direction of the reflected wave is not exactly opposite to that of the incident wave unless the latter was moving at right angles (normal) to the pier. Let f_i, f'_i (Fig. 149) be the wave fronts of a train of waves incident upon the bounding surface BB', f_r, f'_r the front of the same wave after reflection. The waves are supposed to have come from a point so distant that the

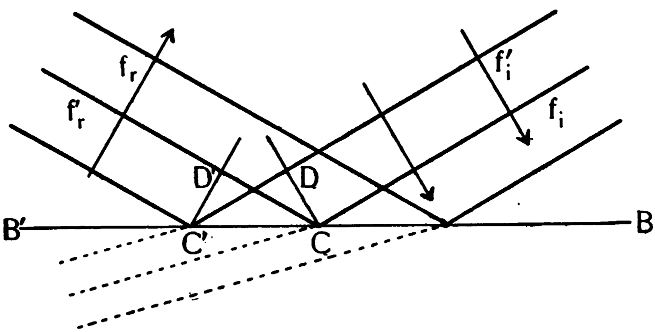


FIG. 149.

wave fronts may be treated as straight lines. When a certain point C on f_i has reached the bounding surface BB', there is a point C' on f'_i which has also just reached BB'. If we draw $CD \perp f_i$ and $C'D' \perp f_r$, it is evident that D will reach C in the same time that C' requires to reach D', whence $DC = D'C' = l$. In the right-angled triangles $C'DC$ and $CD'C'$ the sides $C'D'$ and CD are equal and $C'C$ is common. They are therefore equal and angle $DC'C =$ angle $D'CC'$; that is

to say: *the angles made by the incident and reflected waves with the bounding surface are equal.*

127. Refraction. — In Fig. 150 f_i, f'_i are the fronts of waves incident on BB' and f_R, f'_R the fronts of that portion of the waves which enter the second medium. When f_i has just touched BB' at C, f'_i has also just touched BB' at C', while other portions of these waves,

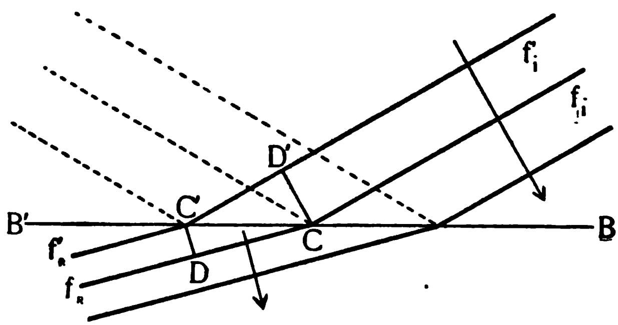


FIG. 150.

f_R, f'_R, have advanced at a different velocity in the second medium. Draw $CD' \perp f'_i$ and $C'D \perp f_R$. The distance $D'C$ was traversed by the wave in the first medium in the same time that the distance $C'D$ was traversed in the second medium, or

$$(35) \qquad \frac{D'C}{C'D} = \frac{v_1}{v_2} = n, \textit{ a constant}$$

The velocity of any particular waves in any given medium is constant and the ratio v_1/v_2 is therefore constant. It is known as the *index of refraction* and is

denoted by n. Refraction may be defined as the change of direction which a wave undergoes in passing from one medium to another. It is evident from the figure that the direction of the wave front will be changed except when the wave front is parallel to the bounding surface, that is, when the angle between the incident wave and the bounding surface is zero. In general, the greater the angle, the more the wave will be refracted.

It should be remarked that reflection and refraction take place in the manner just described only when the bounding surfaces are plane over an area which is large compared with the length of a wave.

The subject of diffraction may well be deferred until light waves are studied.

Exercises.

102. (a) What is the length of the seconds pendulum at a place where $g=979$? (b) What would be the rate of this pendulum at a place where $g=981$?

103. At a certain place a pendulum was found to make 21 oscillations in 20 seconds. Its length was 90.2 cm. What is the value of g at that place?

104. Two pendulums have lengths in the ratio of 3 to 4. What is the ratio of their periods?

105. Place a rectangular dish of mercury or water on the table and strike a series of blows on the table with the fist. Strike simultaneously with both fists at points at right angles to two adjacent sides of the dish and note the waves on the surface of the liquid. A whirling table or an electric motor in motion will often set up such waves.

106. A rope along which impulses are being sent at the rate of 5 per second vibrates in 4 segments. The rope is 6 metres long. (*a*) What is the wave length? (*b*) What is the velocity of the wave?

107. Plot on cross-section paper a train of waves having a wave length of 4 cm. and an amplitude of 1 cm., and a second

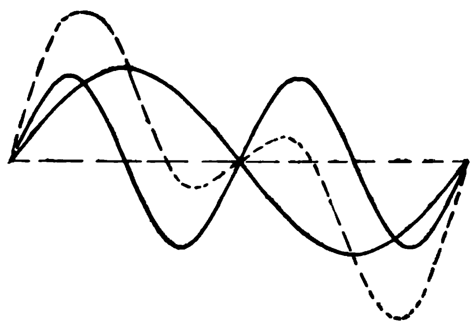


FIG. 151.

train having a wave length of 3 cm. and an amplitude of 1 cm. Plot the sum of the two waves by taking the sum or difference of the distances from the axis of points 2 mm. apart along the axis as illustrated in Fig. 151, where the lengths are respectively 1 and 2.

CHAPTER VII.

SOUND.

128. Nature of Sound. — Sound is a sensation received by the auditory nerve. It has its origin in some vibrating body, as we may easily convince ourselves by observation. If we touch a sounding string or bell we perceive that it is in a state of vibration. Not only so, but when we stop its vibrations by touching it the sound ceases. The vibrations of the sounding body are conveyed to the ear by waves in an elastic medium, usually the air, which sets in vibration the drum of the ear. The way in which the vibrations of the ear drum, after being transmitted by the chain of bones in the middle ear to the fluids of the inner ear, are analyzed and perceived as separate impressions is not well understood. In physics we are more directly concerned with the conditions requisite for producing and transmitting to the ear the vibrations. These are pouring in upon us from every side even in our hours of sleep, and we cannot shut them out as we shut out light. Indeed, the prevention of the vibrations which produce sound must one day become a very important factor in the comfort and health of people who live in cities.

129. Noises and Musical Notes. — The ear delights

in order. A rapid succession of waves of the same sort produces a pleasant impression upon the ear, while a confused mingling of many sorts of waves produces a sound which is disagreeable. The confused rattle of a moving railway train, the squeaking of the wheels against the track as a curve is rounded, and the flapping of the brake shoes against the wheels are not pleasing to the ear of the passenger, but when the train stops at the end of a division and the wheel tester strikes each wheel in turn with his hammer we notice that each wheel gives a clear musical note. We shall find, if we test various bodies like sticks, tables, dishes, stones, iron bolts, indeed any hard body which is free from loose parts, that each body has a definite note of its own.

130. Vibrating Rods. — If we select two wooden rods of equal dimensions we shall find that they give notes nearly alike. By sawing one into two unequal

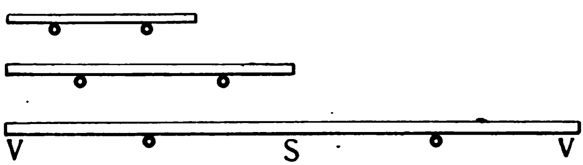


FIG. 152.

parts we shall get two rods which will give notes which are not like either the long rod or each other. Let us see why there should be a difference between them. Let Fig. 152 represent three wooden rods resting on

bits of rubber tubing. They are alike in all respects save length. If a blow be struck at S a wave will travel to V, where it will be reflected and return to S to be again reflected. A series of waves will be sent out into the air at intervals, which depend upon the time taken for the wave to traverse the stick. The ends of the stick are free and are consequently antinodes. There will be another antinode at the centre, with nodes at points one fourth of the length of the stick from each end.

FIG. 153.

The wave length of the stationary wave is the same as the length of the rod. Each part of the rod vibrates like a pendulum, and for rods which are alike in all other respects the period of vibration, like that of the pendulum, varies with the square of the length. A thick rod vibrates more slowly than a thin one of the same length. Metal rods emit notes which are purer and louder than those given by wooden rods. The tuning fork (Fig. 153) is a steel rod bent double and provided with a handle at its middle point. The bending of the fork brings the nodes nearer together than they would otherwise be, and the fork vibrates as indicated in Fig. 154.

FIG. 154.

131. Bells and Plates. — The vibrations of plates are analogous to those of rods, while the vibrations of

bells are somewhat similar to those of forks. When a bell is struck as at A (Fig. 155), its form is altered from the circular to a slightly elliptical shape. Its elasticity draws it back and carries it past, as in other vibrating bodies. The result is to set the bell vibrating in four segments. This may be beautifully shown by filling a jar or glass half full of water and striking the rim. The parts next the antinodes will be thrown into ripples, while the parts next the nodes remain comparatively quiet. If the bell is struck at a point which is slightly heavier than some point near it, this point will tend to become a node and there will be a curious changing effect known as beats. This will be explained in Section 134.

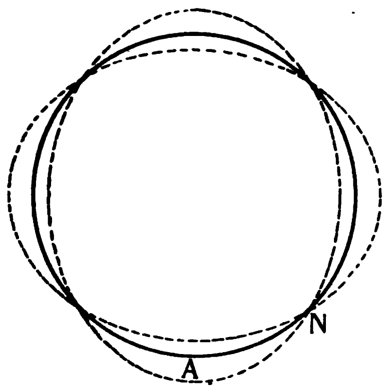


FIG. 155.

132. Pitch. — When two forks of different dimensions are struck in succession we have no difficulty in deciding that the tones they give are different. If one is much larger than the other we shall all agree that the larger gives a tone which differs from that of the small one, just as the tones of a man differ from those of a

child; the long fork has a lower *pitch*, as we say. The pitch of a note is determined by its vibration frequency, n. The pitch of women's voices is, on the average, twice as high as that of men. The pitch of a rod 20 cm. long is four times as high as that of a rod of the same thickness but 40 cm. long. A tuning fork which is not exactly of the proper pitch may be tuned by filing off the ends if it is too low or by filing it thinner if it is too high. The variation of pitch with frequency may be easily shown by holding a card against a toothed wheel which is made to revolve at different speeds, or by directing a blast of air from a tube which has a small nozzle toward a row of holes in a revolving disc of cardboard or metal. Such a disc, called a *siren*, is usually provided with several rows of holes, so that the pitch may be changed by passing the nozzle from one row of holes to another without changing the speed of the disc. If we let the speed of the toothed wheel or siren fall till the frequency is less than about sixteen vibrations per second, we shall begin to hear the separate impulses. When the number of vibrations exceeds 40,000 per second we cease to be able to hear them at all. It is quite probable that certain insects hear notes which our ears are quite unable to perceive because of their high pitch.

133. Musical Intervals. Consonance. Dissonance. — When two musical notes are sounded at once the effect upon the ear is sometimes pleasant and sometimes

very disagreeable. It has been found that this difference depends on the relative vibration frequency of the two notes. The ratio of the frequency of two notes is called the musical *interval* between them. If this interval can be expressed by small numbers the combined effect is pleasant to the ear, and the notes are said to be *consonant.* Intervals which can be expressed only by large numbers are *dissonant.* Primitive music concerned itself mainly with the order of succession of the different notes, the same tune being followed by all voices; modern music combines several tunes or parts in one *harmony*, the variety and richness of which is still further enhanced in the orchestra by employing instruments which give tones of different *quality*. We shall recur to the matter of quality in a later section, but it will be found to be, after all, dependent upon frequency. Vibrations differ only in two particulars — frequency and amplitude. The amplitude of vibration determines the *loudness* of the sound. The remaining characteristics of a musical note, namely, pitch and quality, are dependent upon the rate of vibration.

134. **Beats.** — If two tuning forks have the same pitch they will, when sounded together, produce the most perfect consonance. The interval is $\frac{1}{1}$. If now we attach a bit of wax to a prong of one of the forks its rate will be slightly diminished; the waves from the two forks when sounded together will interfere, first reenforcing, then destroying each other. The effect pro-

duced is called a *beat*. It is disagreeable, producing much the same effect upon the ear that a flickering light does upon the eye. The number of beats per second is equal to the difference between the vibration numbers of the two notes. Tuners of musical instruments make use of beats in tuning the instrument. They have only to seek to diminish the number of beats until beats are no longer heard; when the string or pipe is known to be in unison with the standard fork. Beats in bells are overcome to some extent by chipping away with a chisel the parts which are too heavy.

135. Overtones or Partials. — We saw in Section 125 that a body of limited dimensions, like a string with its ends fastened, may vibrate as a whole or in any whole number of equal parts. It has been found that most vibrating bodies when vibrating as a whole also vibrate in segments at the same time. When a body vibrates as a whole it gives the lowest note which it is capable of giving. This is called its *fundamental* tone. The highest tones which it gives when vibrating in parts are called *partials* or *overtones*. When it vibrates in two parts it gives its first overtone, in three parts its second overtone, and so on. The first five partial tones are perfectly consonant with the fundamental and each other, for their intervals are all expressed in small numbers: $\frac{2}{1}$, $\frac{3}{1}$, $\frac{3}{2}$, $\frac{4}{1}$, $\frac{4}{2}$, $\frac{4}{3}$, etc.

The overtones given by any particular sounding body will depend much upon the shape and material of the

body itself. Since the overtones present in one voice or instrument are different from those in another, the tone produced by their combination with the fundamental will be different. The pitch of the note we judge from the pitch of the fundamental, which, if present at all, is usually the prevailing note. The minor differences produced by the peculiarities of the instrument we call the quality of the note. Strings break very readily into segments and are therefore rich in overtones. A board can vibrate in almost any number of parts, yet has no marked preference for its fundamental. Boards are therefore well adapted to take up the vibrations of strings and forks which are in contact with them, thus reënforcing the note of the string or fork.

136. Resonance. Pipes. — If one lifts the dampers from all the strings of a piano and sings a single note, the strings corresponding to the fundamental and the principal overtones of the note sung will be set in vibration and may be distinctly heard. A window in a church will often be set rattling when a particular note of the organ is sounded. This phenomenon is called *resonance*. It is due to the fact that the resounding body has the same natural period of vibration as the sounding body, so that the impulses given by the sounding body are timed just right to reënforce each other. The reënforcement of a tuning fork by a column of air is easily shown by holding a fork which has just been struck over a tall jar and slowly pouring water into the

jar to shorten the air column. When the air column is exactly the right length the fork will speak loudly enough to be heard distinctly all over the room (see Fig. 156). The bottom of the pipe will be a node and the top will be an antinode, so that the length of the column would be exactly $\frac{1}{4}l$ if it were not for the fact

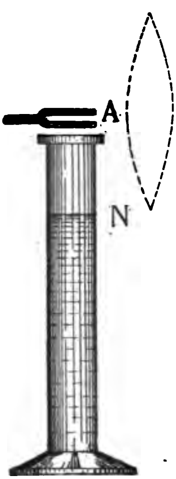


FIG. 156.

that the size and shape of the opening at the top changes slightly the position of the antinode, throwing it a little distance above the top of the jar. If a jar three times as tall as the shortest column of air which will reënforce the fork is used resonance will again occur, for the air column will break into segments, as shown in Fig. 157. The resonance of air columns is employed in organ pipes, the pipes being made of the proper length to reënforce the different notes. The air is set into irregular vibrations by being blown against the mouthpiece in Fig. 158. Those vibrations which are of the right period to be reënforced by the pipe are the ones which we hear. The closed pipe, like the resonance jar, is one fourth the length of a wave in air. The open pipe (Fig. 159) has

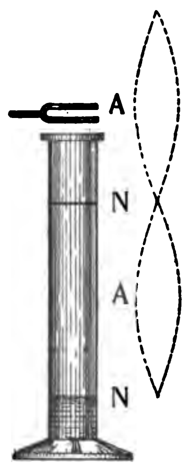


FIG. 157.

an antinode at each end. Its length, therefore, is half a wave length, hence it must be twice as long as a closed pipe which is to give the same note.

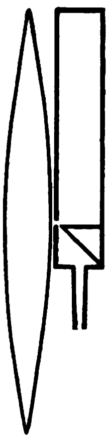


FIG. 158.

Organ pipes are often provided with reeds like the reeds of an accordion or harmonica (mouth organ). Organ pipes are often lacking in some of the higher overtones. The lack is made good by smaller pipes which are so connected to the keys as always to speak when the fundamental does.

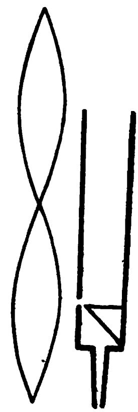


FIG. 159.

Such wind instruments as the flute and cornet have means of varying the length of the air column to produce a number of different notes. Each of the fundamental lengths can be made to serve for a number of notes by varying the tension of the lips and the pressure of the breath, as is done in the bugle. The fundamental note of such instruments is not used.

137. Velocity of Sound Waves.—The resonance jar furnishes a convenient method of measuring the velocity of sound in air. Thus in Fig. 160

$$(34) \quad \overline{NN} = \tfrac{1}{4}\, l$$

$$(33) \quad v = nl$$

whence: $$(36) \quad v = 2n\, \overline{NN}$$

where the vibration number of the fork is supposed to be known. The velocity of sound varies inversely with the square root of the density of the medium. Since, therefore, air expands by heating, sound travels faster in warm air than in cold. The velocity of sound in air at 0° C. is 332 metres per second. At any temperature, t, it is

$$(37) \quad v = 332\sqrt{1 + .004\, t}$$

since air expands $\frac{1}{273} = .004$ for each degree Centigrade above 0° C.

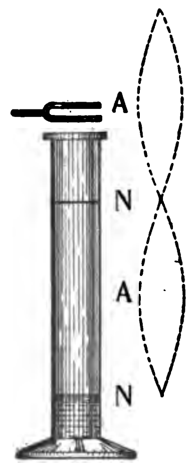


FIG. 160.

The velocity of sound in solids may be found by a beautiful method due to Professor Kundt. He clamps a rod at its middle point and attaches a cork to one end, which is allowed to project into a long glass tube (see Fig. 161). The bottom of the tube as it lies in a horizontal position is covered with fine cork filings, and the other end of the tube is closed with a movable cork which fits snugly and is used to

FIG. 161.

vary the length of the tube. The rod is stroked with a rosined cloth and thus set into rapid longitudinal vibrations. The cork, which fits loosely in the tube, sets the air in the tube in vibrations, which by reflection from the farther end produce stationary waves in the tube.

The dust at the antinodes is violently agitated and falls into little ridges when the sound ceases. The distance between the antinodes is easily measured. The middle of the rod is a node and the ends antinodes. The length of the rod is, therefore, half the length of a wave in the solid. If v_a is the velocity of sound waves in air at the given temperature, then v_s, the velocity in the solid used, is

$$(38) \quad v_s = \frac{l_s}{l_a} v_a$$

where l_s, l_a are the lengths of a wave in the solid and in air respectively.

It should be borne in mind that all stationary waves are caused by the interference of waves reflected from

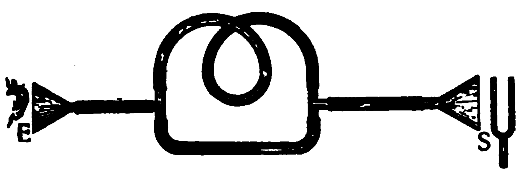


FIG. 162.

opposite bounding surfaces. The length of a wave may be measured, also, by the interference of two waves from the same point, which travel by different paths to a second point, as shown in Fig. 162. If one branch of the rubber tube is half a wave length longer than the other for waves of the frequency of the fork which is vibrating at *S*, the waves will meet at *E* in

opposite phase and produce silence. An interesting example of the production of silence by the interference of two sound waves is the case of a tuning fork. If a vibrating tuning fork be slowly rotated near the ear or over a resonance jar, four positions may be found where no sound is heard.

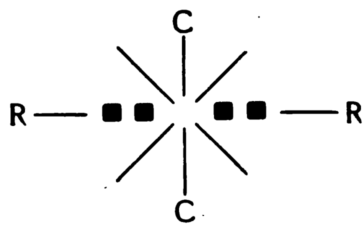


FIG. 163.

If while the fork is in one of these positions a paper tube be slipped over one prong of the fork so as not to touch it, the sound will again be heard. This is readily understood by referring to Fig. 163. When the prongs approach each other condensations are sent toward *C*, rarefactions toward *R*. At all points on the diagonal lines, which are equally distant from *C* and *R*, the waves will meet in opposite phases and destroy each other.

138. Vibrating Strings. — We have referred so often to the vibrations of stretched strings that the nature of these vibrations should now be pretty well understood. The mathematical laws governing the vibration frequency of strings are easily demonstrated by experiment. They may be stated as follows:

The number of vibrations per second of a tightly stretched string which is giving its fundamental note is:

1. Inversely proportional to its length.
2. Directly proportional to the square root of the tension.
3. Inversely proportional to its linear density.

These three laws may be summed up in one formula:

$$(39) \quad n = \frac{1}{2L}\sqrt{\frac{f}{m}}$$

where L is the length of the string, f is the tension in dynes, and m the mass per unit length of the strings.

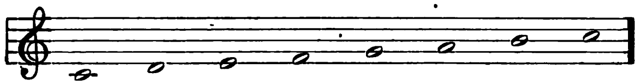
These laws are all exemplified by such stringed instruments as the guitar and the violin. The player raises the pitch of any string by touching it so as to shorten the vibrating portion. He raises the pitch in tuning by increasing the tension. The heavy strings are the ones which give the lower notes.

The point at which a string is struck or bowed affects its quality by determining to some extent what overtones shall be present. The hammers of a piano are so placed as to prevent the occurrence of a certain higher overtone which is not consonant with the others. Overtones are often spoken of as *harmonics*. This term should be applied only to such overtones or partials as are consonant with the fundamental tone.

139. Musical Scales.—The foundation of all our music is the scale of eight notes, the eighth or *octave* of which has just double the frequency of the first. The *relative* frequencies of the major scale are variously

represented in lines 1, 2, and 3, the *absolute* frequencies in lines 4, 5, and 6.

1.	1/1	9/8	5/4	4/3	3/2	5/3	15/8	2/1
2.	First	Second	Third	Fourth	Fifth	Sixth	Seventh	Octave
3.	Do	Re	Mi	Fa	Sol	La	Si	Do

4. 

5.	c′	d′	e′	f′	g′	a′	b′	c″
6.	264	297	330	352	396	440	495	528

This scale is made up by combining three *major triads*, and is therefore known as the *major scale.* The major triad is so called because it is composed of three notes which, when sounded together, give the most perfect harmony, namely, the first, third, and fifth, or, Do, Mi, Sol. If we add the octave it gives us the major chord. Helmholtz pointed out that the reason for the perfect consonance of the major triad lies in the fact that no beats occur between any of the overtones of any tone either with the fundamentals or overtones of any other of the triad. This may be illustrated by an example.

	FIRST	THIRD	FIFTH
Fundamental	200	250	300
First partial	400	500	600
Second ,,	600	750	900
Third ,,	800	1,000	1,200
Fourth ,,	1,000	1,250	1,500
Fifth ,,	1,200	1,500	1,800

It will be seen that there is unison between the second partial of the first and the first partial of the fifth, as also between the fourth partial of the first and the third partial of the third. Indeed, there are four pairs in unison. But in no case is the difference between the number of vibrations of any pair less than the difference between the fundamentals of the first and fifth. By way of contrast let us compare the second, fourth, and sixth.

	SECOND	FOURTH	SIXTH
Fundamental	225	267	333
First partial	450	534	666
Second ,,	675	801	999
Third ,,	900	1,168	1,332
Fourth ,,	1,125	1,335	1,665
Fifth ,,	1,350	1,582	1,998

The combination 666 and 675 is bad, but when we combine 1,350, 1,335, and 1,332 the effect is wholly unpleasant.

The relative frequencies of the major triad, as will be seen from the table, are:

$$\begin{array}{ccccc} 1/1 & : & 5/4 & : & 3/2 \\ c' & : & e' & : & g' \end{array}$$

or, expressed in whole numbers:

$$\begin{array}{ccccc} 4 & : & 5 & : & 6 \\ c' & : & e' & : & g' \end{array}$$

If we form a second triad taking c″ as 6, and still another taking g′ as 4, we have:

$$\begin{array}{ccccc} 4 & : & 5 & : & 6 \\ c' & : & e' & : & g' \\ f' & : & a' & : & c'' \\ g' & : & b' & : & d'' \end{array}$$

From which we obtain:

$f' : c'' = f' : 2c = 4 : 6$ whence $f' = 4/3\ c'$

$a' : c'' = a' : 2c = 5 : 6$ whence $a' = 5/3\ c$

$g' : b' = 4 : 5$ and $g' : c' = 6 : 4\ \therefore\ b' = 15/8\ c'$

The major scale is thus seen to be composed of three major triads.

It is readily seen on inspecting the table of intervals below that there are five intervals of nearly equal magnitude and two which are only about half as large. The former are called *full tones*, the latter *semitones*.

Note,	c'		d'		e'		f'		g'		a'		b'		c''
Relative frequency,	1/1		9/8		5/4		4/3		3/2		5/3		15/8		2/1
Intervals,		9/8		10/9		16/15		9/8		10/9		9/8		16/15	

To make possible the writing of music in scales higher than the one just shown (which is known from its keynote as c major) semitones have been inserted between c' and d' and in the other intervals, which are full tones, by sharping or flatting the notes — that is, by raising them $\frac{1}{24}$ or by lowering them $\frac{1}{25}$. Thus: c♯ (c sharp) is equal to $\frac{25}{24}$ c, while d♭ (d flat) is $\frac{24}{25}$ d. The semitone added is not halfway between c and d, and c♯ and d♭ are not identical, for c♯ $= \frac{25}{24}\ c = \frac{25}{24} \cdot 264 = 275$, while d♭ $= \frac{24}{25}\ d = \frac{24}{25} \cdot 297 = 285$.

In ascending the scale sharps are used, in descending flats are used. The scale of eighteen notes thus formed is called the *chromatic scale.*

The *tempered scale* was devised for use with instruments like the piano and organ, which, unlike the violin and the voice, must have a separate string or pipe of fixed length for each note. To avoid the excessive number of notes required the octave is divided into twelve equal intervals, each of which has a frequency equal to $\sqrt[12]{2} = 1.059$ times the note below. This arrangement necessitates throwing all intervals except the octave slightly out of tune. Long use makes us accustomed to the lack of perfect harmony, so that we give little heed to it. The human voice can hardly be at its best, though, when accompanied by the piano.

The progress of music as an art side by side with the science of music is a good example of the interdependence of all human efforts.

Exercises.

108. (*a*) What length of closed organ pipe will give c′? (*b*) What note will an open pipe of the same length give?

109. (*a*) Compute the linear density of a string of a monochord (Fig. 164) after measuring its diameter. Make the other measurements necessary for computing *n* in formula (39). (*b*) Calculate *n* by means of a resonance tube made of a large glass tube with a piston for varying its length, and see how the two values agree.

110. What is the velocity of sound waves in a brass rod 120 cm. long which excites waves 24 cm. long in a Kundt's tube? The temperature of the air was 20° C.

111. (*a*) Will a tuning fork vibrate longer in the hand or resting on a table? (*b*) In which place will it give the louder sound? Explain.

112. An echo is sent back (by reflection) from a barn. The last of a series of words requiring five seconds to utter can be spoken just before the first word is returned. The air is at the temperature of freezing water. How far away is the barn?

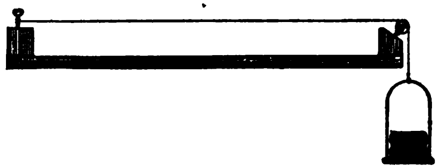


FIG. 164.

113. Two telegraph sounders are on the same circuit, which is closed twice a second by a pendulum. One sounder is by the observer's side, the other is 550 feet away. What is the velocity of sound if the two sounders seem to tick in unison?

114. A stone is dropped down a well. After two seconds the splash of the stone in the water is heard. How far is the surface of the water below the top of the well?

115. (*a*) An organ pipe gives a note of frequency 264 at 16° C. What will it give at 28° C.? (*b*) Would a piano string which is in tune with the pipe at 15° be in tune at 28° C.?

116. A tuning fork placed at *S* (Fig. 162) gives the minimum sound at *E* when the long arm of the tube is 65 cm. longer than the short arm. The temperature is 20° C. What is the pitch of the fork?

CHAPTER VIII.

LIGHT.

140. The Sensation of Light. — Light is a sensation which is peculiar to the optic nerve. It may be produced in various ways, but is usually the result of disturbances in external objects which are transmitted to the eye as waves. What we have learned of waves in general and of sound waves will be of use to us in the study of light. It is common to call by the name light the cause of the sensation of light as well as the sensation itself, and no serious confusion need result from our using the term in both senses.

141. Definitions. — Every body which is at any instant visible to us is said to emit light at that instant. A small class of bodies emit light regardless of the presence or absence of other luminous bodies. Such bodies are said to be *self-luminous*. They are usually, though not always, very hot, like the sun or a gas flame. Bodies through which objects are clearly visible are *transparent*, as water and glass. Bodies which are not transparent but yet allow much light to pass through are *translucent*. Such are milk, ground glass, opal. Bodies which allow no light to pass are *opaque*. Most solid bodies (not crystals) are of this class.

142. Light from a Point. Shadows. — Every luminous body may be thought of as made up of a large number of luminous points. If light is a disturbance which spreads from a luminous point in a uniform medium it will advance with equal velocities in all directions until it meets a different medium. Let S (Fig. 165) be such a luminous point. The wave fronts are concentric spheres and are always perpendicular to radii drawn from S as a centre. An opaque object as OO' cuts off all that portion of the waves which strikes it, leaving a space $POO'P'$ which is not illuminated by S. This agrees with what we have all observed. It is commonly expressed by saying that light travels in straight lines. The normal to a wave front is often called a "ray." We shall hereafter understand the word ray in that sense.

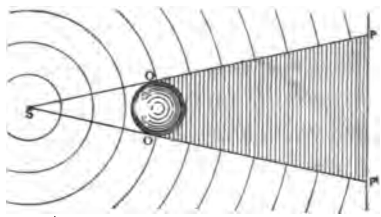


FIG. 165.

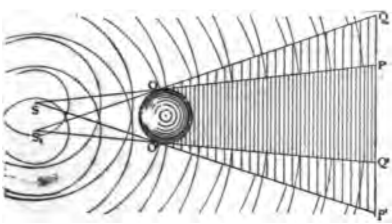


FIG. 166.

The shadow cast by a luminous surface has not sharp outlines, since there are portions which lie in the shadows of some points and yet are illuminated by other points. Fig. 166 shows the shadow cast by a luminous

surface. There are supposed to be an unlimited number of points between S and S_1. The space between P and Q receives light from an increasing number of these points as we proceed from P to Q. The space between P and Q' receives no light from SS_1. It is called the *umbra*, or full shadow. The rest of the space between Q and P' is called the *penumbra*, or partial shadow. During a partial eclipse of the sun that part of the earth where the eclipse is seen passes into the penumbra of the shadow cast by the moon. During a total eclipse the observer is in the umbra.

143. Pictures by Small Openings. — A beautiful illustration of the rectilinear propagation of light is furnished by making a single small opening through the side of a box and fastening a sheet of paper against the side opposite the opening.

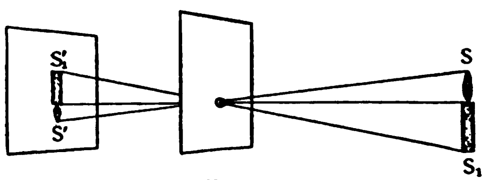


FIG. 167.

If we follow the light from two points of an object, as SS_1 (Fig. 167), till it strikes the paper, we see that for every point of the object there is a corresponding spot on the paper. The result is to give us, on the paper, an inverted picture, or *image* as it is called, of the object. The image is, however, faint if the hole is very small, and blurred if the hole is larger, for the spots $S'S_1'$ are not points, but circles larger than the opening.

All our perception of the forms of objects is conditioned upon the formation of images upon the sensitive wall (retina) of the eye. We shall now explain the ways in which images may be formed which are much more perfect in detail as well as more brilliantly illuminated than the pinhole images just described.

144. Pictures by Reflection. — We learned in our study of waves that a wave after reflection from a

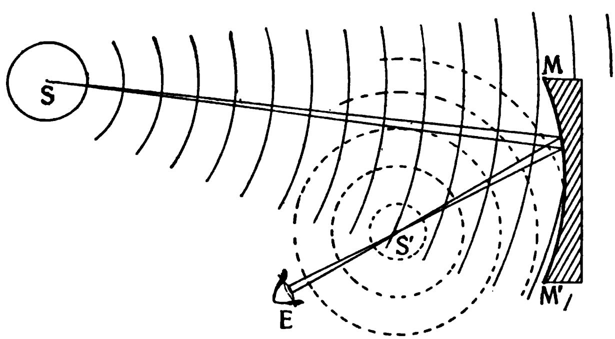


FIG. 168.

plane surface makes an angle equal to the angle of incidence, but in the opposite direction. Every reflecting surface is plane for a very small area at any point on the surface. The angle made by a wave at any point on a curved surface with the surface at that point is the angle which it makes with a plane tangent to the surface at the point. Let us bear in mind that the difficulty with our pinhole image is that the light from S which falls exactly at S' (Fig. 167) is only that which travels along

the radius SS', and if we attempt to get more light by making the hole larger, none of the additional light admitted falls exactly upon S', but only near it, making a confused image. The problem is, then, to make a large portion of the waves which diverge from S meet at S'. This can only be done by changing the direction of the waves in some regular manner, such that those which diverge most from the radius SS' shall be bent

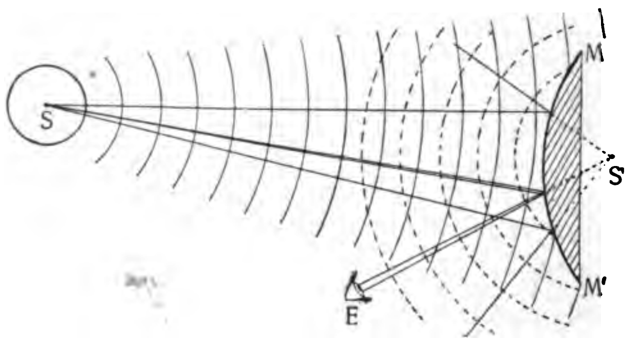


FIG. 169.

more than those which diverge less. This is accomplished if the waves are reflected from a spherical surface MM', as in Fig. 168, where all the waves from S which strike the mirror meet at S', which is called the image of S. The light after reflection seems to an eye at E to diverge from S'. A series of such images arranged symmetrically with corresponding points in the object constitute a *real* image of the object. When the mirror is of such a form that the waves do not

really diverge from centres, but only seem to do so, the image is said to be *virtual.* Thus the convex mirror in Fig. 169 makes the waves from S diverge still more than they did before striking the mirror, but they seem to diverge from a point S' behind the mirror where there are really no waves from S, and S' is said to be a virtual image of S.

A plane mirror also gives a virtual image, but the

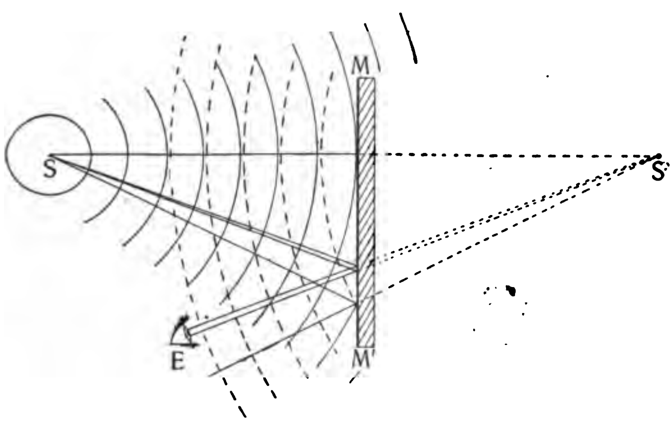


FIG. 170.

amount of divergence of the waves is unchanged (see Fig. 170). It is easily shown that the image of a point in a plane mirror lies in a line drawn from the point perpendicular to the mirror and as far behind the mirror as the point is in front. Let S' (Fig. 171) be the image of S in a plane mirror, and SP a perpendicular to the mirror. S' must lie on the line SP pro-

duced, since SP is at once perpendicular to the mirror and the wave front, making the angle of incidence zero. Let SQ be the direction of another portion of the wave which strikes the mirror at Q and draw RQ perpendicu-

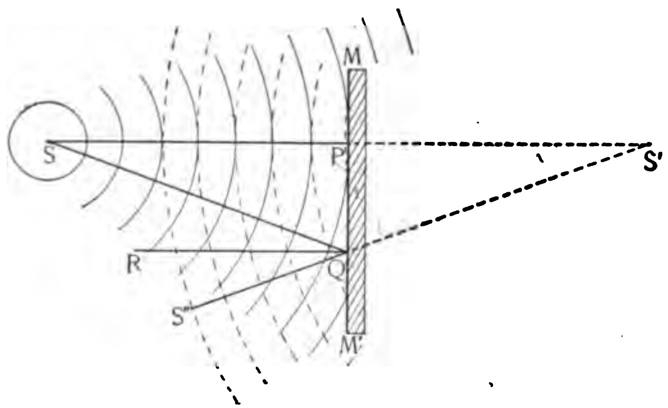


FIG. 171.

lar to the mirror at Q. Let QS'' be the direction of propagation of the wave after reflection from Q.

$$
\begin{array}{rll}
& \text{Angle } PSQ & = \text{Angle } SQR \\
& \quad " \quad SQR & = \quad " \quad RQS'' \\
& \quad " \quad RQS'' & = \quad " \quad PS'Q \\
\text{whence:} & \quad " \quad PSQ & = \quad " \quad PS'Q
\end{array}
$$

In the right triangles SPQ and $S'PQ$ the sides PQ are identical, and angles PSQ and $PS'Q$ are equal. The triangles, therefore, are equal and $PS' = PS$.

145. Position of the Image of an Object. — To determine the position of the image of an object we may

locate the image of two or more prominent points. In the case of plane mirrors this is easily done by drawing perpendiculars through the mirror to points as far behind the mirror as the luminous points are in front of it. In the case of spherical mirrors it might be done by drawing any two radii till they meet after reflection. The measurement of angles is, however, less easy than the measurement of lines, and it happens that there are two radii (or rays as they are commonly called) which can always be drawn without measuring the angles. Let MM' (Figs. 172 and 173) be a spherical mirror drawn from C as a centre. A line through C perpendicular to the centre of the mirror is called the principal axis of the mirror.

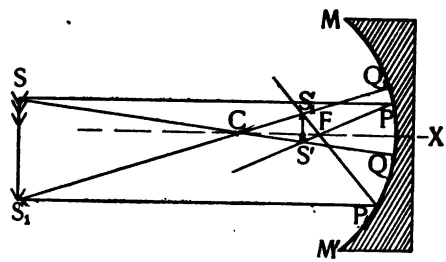


FIG. 172.

All waves passing along a ray drawn through C are reflected back along the same ray. It can be proved that all waves moving along rays drawn parallel to the principal axis, CX, will be reflected along rays which pass through a point F, halfway between C and the mirror. This point is called the principal focus.* It is the place where plane waves, like those from the sun or any distant object, meet after reflection. Let it be required to find the image of an

* Latin *focus*, fireplace.

object SS_1. Draw SPF, making SP parallel to CX. Draw SCQ. Their point of intersection S' is the image of S. Find the image of S_1 in the same manner. Join them and we have the image of the object. For different positions of the object the position of the image will be different, but the method of finding the position of the image is always the same.

The statement that all rays parallel to the principal axis pass through the principal focus is only true if the mirror is a small segment of a sphere. Large mirrors

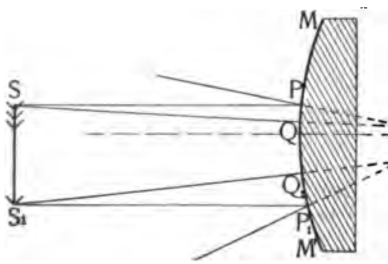


FIG. 173.

must have a correspondingly large radius of curvature. Spherical mirrors are but little used in optical instruments, the reflecting telescope being almost the only example of their use. They are much employed for collecting and directing the light of signal lamps, locomotive headlights, and the like, where no accurate image is required. If the lamp is placed at the focus of a concave mirror the reflected rays will be parallel. By moving it toward the mirror any desired amount of divergence may be produced. By moving it away from

the mirror the light may be made to converge at any desired point.

146. Pictures by Refraction. — We have seen in Section 127 that, in general, waves suffer a change of direction in passing from one medium to another. Let *LL* (Fig. 174) be a segment of a sphere, of glass say, made by a plane passing at some distance from the centre, so that it shall be but a small segment of a sphere, or by the intersection of two spheres. It is a *lens*. When convex on both sides, it is called a double-convex lens. Light waves from *S* which fall upon *LL* (Fig. 175) will be in part reflected, but we are at present concerned with the part that passes through the lens. The waves strike the central part of the lens first, and are retarded, since

FIG. 174.

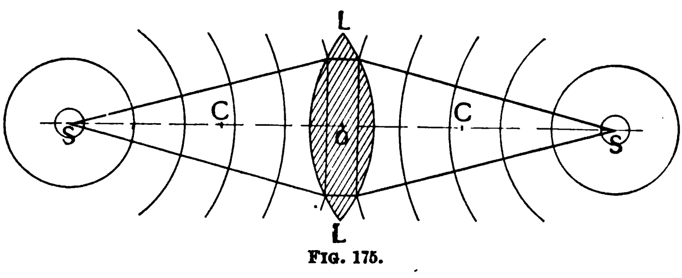


FIG. 175.

the velocity in glass is less than in air. In the case shown in Fig. 175 the waves in the glass are nearly plane. The portions of the wave farthest from the axis, *CC*, emerge first and gain upon the part that has not

yet emerged, so that when the wave is again in air its curvature has been reversed, and it converges at S'.

All lenses which are thicker at the centre than at the edge converge the waves and are called converging lenses. Three kinds of converging lenses are shown in Fig. 176, namely: (1) double-convex, (2) plano-convex, (3) concavo-convex.

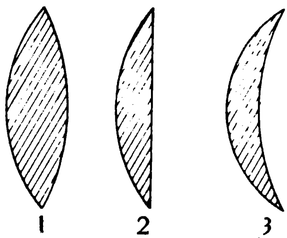


FIG. 176.

The *principal focus* of a converging lens is easily found by experiment. It is the place of convergence of plane waves which have passed through the lens. In a double-convex lens of flint glass with equal curvatures on its two sides, the focus is not far from the centre of curvature. The distance OF (Fig. 177) is called the *focal length* of the lens.

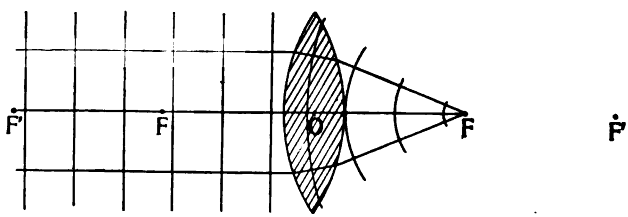


FIG. 177.

An object placed at a distance $2\,OF$ from the lens will be focused at an equal distance on the opposite

side at F', which is sometimes called the secondary focus.

Objects outside F' will be focused between F and F', and objects between F and the lens will be focused outside F'.

Lenses which are thicker at the edges than in the centre increase the divergence of waves and are called diverging lenses. Since they cannot cause divergent waves to converge, they never form real images, but only virtual ones (see Fig. 178).

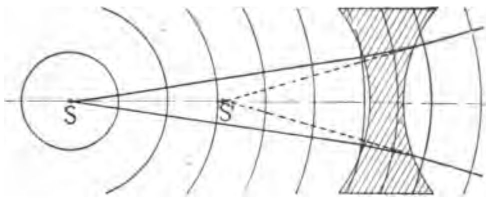


FIG. 178.

Three forms of diverging lenses are shown in Fig. 179, namely: (1) convexo-concave, (2) plano-concave, (3) double-concave.

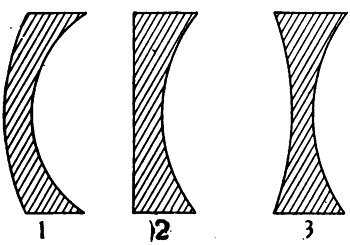


FIG. 179.

Diverging lenses are used by draughtsmen to enable them to judge how a drawing will appear when reduced in size. They are used also in spectacles for near-sighted persons. Most lenses used in optical instruments are converging.

147. Position of Images Formed by Lenses. — As in mirrors, so in lenses, if we find where two rays from

a point meet again we have found the image of that point.

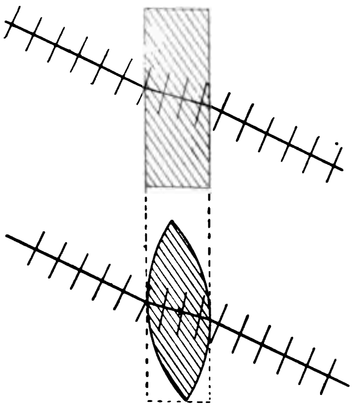


FIG. 180.

Rays parallel to the principal axis are refracted so as to converge at the focus. Waves which pass through O, the optical centre of the lens, suffer no permanent change of direction, but only a slight displacement to one side, like light which has passed through a plate with parallel sides (see

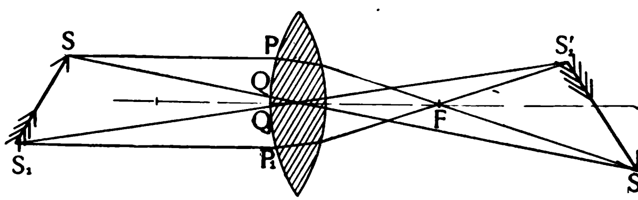


FIG. 181.

Fig. 180), for the faces at points intersected by a line through the centre are parallel.

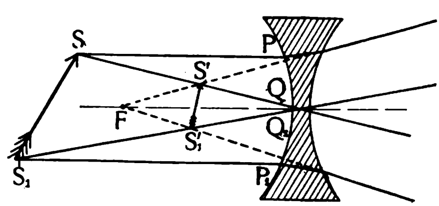


FIG. 182.

To find the

image of an object, SS_1 (Figs. 181 and 182), we first find the image of S by drawing rays SPF and SQO and producing them till they meet at S'. The image of S_1 is found in a similar manner. The image is found to be real and inverted in one case, virtual and upright in the other.

The two cases shown are typical and all other cases may be treated in the same manner.

148. Some Optical Instruments. — The simplest optical instrument is the magnifying glass, or *simple microscope.* It consists of a converging lens. The object is placed between F and the lens, and the image is virtual, upright and enlarged, as shown in Fig. 183.

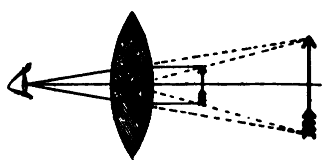


FIG. 183.

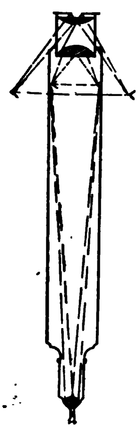


FIG. 184.

The *compound microscope* (Fig. 184) consists of a small converging lens of short focus, called the objective, which is placed near the object and forms an inverted real image at the upper end of the tube, and

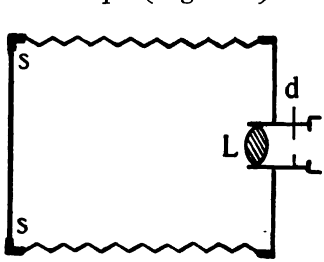


FIG. 185.

a second converging lens called the eyepiece, which is a single or double magnifying lens used to view the image formed by the objective.

The *camera* consists of a dark box having a lens, *L* (see Fig. 185), at the front which forms a real inverted image on a screen at the back. The distance between lens and screen, *ss*, may be varied to bring objects at different distances into focus. The screen may be removed also, and replaced by a sensitive plate. A diaphragm, *d*, often of the form shown in Fig. 186, may be turned so as to control the amount of light admitted through the lens.

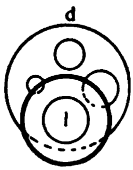


FIG. 186.

The *eye* is very like a camera. It has a lens called the crystalline lens, *L* (Fig. 187), a sensitive membrane, the retina, *r*, at the back to receive the image, a diaphragm, the iris, *i*, for varying the amount of light, and an arrangement for adjusting the focus for near or distant objects.

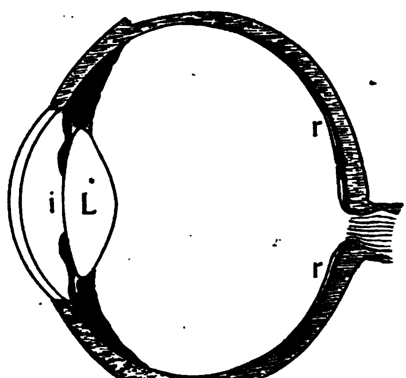


FIG. 187.

This adjustment is called accommodation, and is accomplished by changing the curvature of the lens by means of certain muscles at its circumference.

Many eyes are faulty in having the retina too near to or too far from the lens. When the retina is too far back the object must be brought near in order to be distinctly seen. The person is near-sighted and should wear concave glasses. When it is too near the person is far-sighted and should wear convex glasses.

When the normal eye is at rest it is in focus for distant objects. Near objects are seen only by bringing into use the muscles of accommodation. At about forty years of age we begin to lose the power of accommodation and must wear glasses for reading, though distant objects are still seen distinctly.

There are other errors of vision which the skilled oculist can correct. One who has any difficulty with

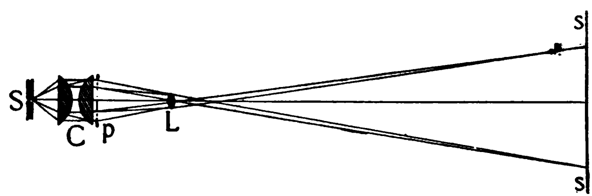


FIG. 188.

his eyesight should consult an oculist without delay. Errors of vision cause headache and may, if uncorrected, lead to inflammation of the eye and serious impairment of vision.

The *magic lantern*, or stereopticon, consists of a pair of lenses called the condenser, C (Fig. 188), the purpose of which is to collect the light from a bright

source, S, like an arc lamp, and make it converge upon a projection lens, L. Between C and L and very close to the former is placed, inverted, the transparent picture, p, an image of which is thus projected upon a large screen at ss.

COLOR.

Up to this point we have considered only the *forms* of images. We all know, however, that the *color* of an object has often as much to do with its appearance as the form. The image formed in the camera is a colored image, though the ordinary photograph reproduces only the lights and shades of the picture and not its color.

The eye, unlike the photographic plate, distinguishes differences in the appearance of objects which have exactly the same shape, as when ripening fruit or autumn leaves change from green to red.

According to Helmholtz, the eye is sensitive to three fundamental impressions, namely, red, green, and violet. The hundreds of colors which we are able to distinguish result from the blending of these three fundamental sensations in varying proportions. What we call white light is the mixture of equal parts of the three.

149. Color by Refraction. The Spectrum. — The compound nature of white light was shown by Newton in a beautiful experiment which may easily be repeated by any one. Sunlight is admitted to a darkened room

through a small hole in a shutter and allowed to fall upon a prism of glass, as shown in Fig. 189. The light will be refracted, as we know already, but it will not all be refracted to the same extent. The result is to give us a row of images of the opening, forming a beautiful band of colors called a *spectrum.*

If the opening is circular the images will overlap and the colors will be mixed, but if a narrow slit is used the colors are pure.

The *rainbow* is a great spectrum produced by the

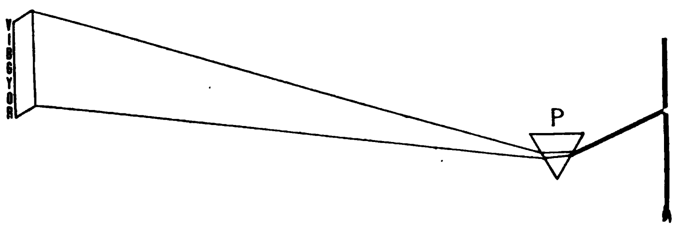


FIG. 189.

refraction of light in drops of water. Close observation will show that the arrangement of the colors is the same as in Newton's spectrum.

It is now known that the difference in light waves which causes a difference in refrangibility and also in color is one of wave length. The unit of wave length is the millionth of a millimetre. The shortest visible waves are the violet, 380 units long, and the longest are the red, reaching to 688. Between these limits are waves of every possible length.

The violet sensation is excited most by waves in the neighborhood of 450, but as far as 380 on the one side and 560 on the other. The relative effect of the different wave lengths in producing the violet sensation is shown by the height of curve *V* (Fig. 190).

The green sensation is excited most strongly by 550, but responds to waves from 440 to 640 (see *G*).

The maximum effect of the red is at 570, but it extends as far as 480 and 688 (see *R*).

It will be seen that the waves between 480 and 560 excite to some extent all three of the primary sensations in the retina of the eye. The effect of exciting the violet and the green at the same time is to give us the sensation of blue, which may be a deep indigo if but little green is present, or an emerald if but little violet is present.

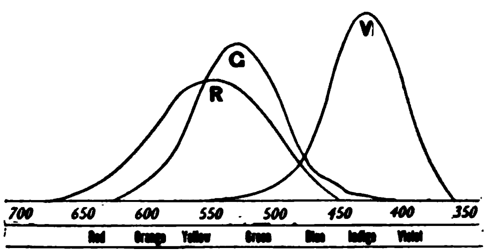


Fig. 190.

In the same way yellow and orange result from mixing the sensations of green and red.

The mixture of red and violet gives purple, a color which does not appear in the spectrum, since there are no waves which excite both red and violet, and not green.

Any pure *hue* mixed with white gives *tints;* mixed

with black, *shades*. Thus the browns are shades of orange and sky blue is a tint of blue.

150. Color Mixture. — If a circular disc be divided into three equal sectors, each of which is painted one of the primary colors, red, green, violet, as in Fig. 191, the disc will show, when rapidly rotated about its centre, no one of these colors, but will appear gray. If the colors were perfectly pure it would appear white, but the black in them gives the mixture a gray appearance. A smaller disc having white and black segments may be placed over the colored disc, and, if the black and white are in the right proportions, the two grays will match.

The reason that the separate colors are not seen when the disc is in motion is that any impression produced upon the retina persists for a fraction of a second, so that each of the colors in the rotating disc affects the retina the same as if the others were not present. The result is that three colored images fall upon the same spot of the retina, and, since these colors excite all the sensations that white light is able to excite, and in the same proportions, the effect produced is that of white light. If the colors were as bright as those of the spectrum the mixture would be white. Since the colors are all more or less mixed with black, it is in reality gray.

A convenient plan for mixing the colors in any desired proportions is to make a number of discs like the one shown in Fig. 192, and paint each a different color. The discs have each a radial slit so that they may be

slipped one upon the other, as shown in Fig. 193. They may then be turned so as to expose to view any desired proportions of the different colors.

Another method of mixing the sensations is to rule alternate lines of the colors very close together, as in Fig. 194. The images are not kept distinct by the eye, but blend together and give the composite color, orange or blue.

Experiment shows that a mixture of red and greenish blue produces gray. This should be so, since blue is composed of green and violet. Similarly, yellow and indigo blue produce gray, since together they contain all the primary colors.

Such pairs of colors are called complementary colors. A number of such pairs are shown in the following table: —

COLOR	COMPLEMENTARY
Red	Greenish blue
Orange	Blue
Yellow	Indigo
Green	Purple
Bluish green	Red
Blue	Orange
Indigo	Yellow
Violet	Greenish yellow

The colors made by mixing pairs of spectrum colors are shown in the following table, where the mixture of any color named in the top row with any color in the left-hand column will be found at the intersection of the corresponding row and column:

[illegible]

Fig. 192.

Fig. 193.

	Violet	Indigo	Blue	Blue Green	Green	Yellow
Red	Purple	Dark Rose	Light Rose	White	Light Yellow	Orange
Orange	Dark Rose	Light Rose	White	Light Yellow	Yellow	
Yellow	Light Rose	White	Light Green	Light Green	Green Yellow	
Green Yellow	White	Light Green	Light Green	Green		
Green	Light Blue	Green Blue	Blue Green			
Blue Green	Green Blue	Green Blue				
Blue	Indigo					

151. Colors by Absorption. Pigments.—What we call the color of an object is the color which it shows when illuminated by white light. The reason that different objects have different colors is that they do not reflect all colors in the same proportion.

When white light falls upon a red brick wall the waves which produce green and violet sensations are absorbed by the brick and converted into heat, while the waves which produce the red sensation reach the eye. In like manner an orange absorbs the violet and reflects green and red in about equal proportions, while a lemon absorbs some of the red as well as the violet, giving us a greenish yellow.

A black object is one which absorbs all the waves,

while a gray object is one which absorbs about the same part of each of the primary colors.

We may now understand why the colored discs produce gray and not white. It is because the red disc absorbs not only the green and violet, but a considerable portion of the red as well. The same being true of the other colors, the mixture must contain a large amount of black.

When two paints are mixed the result is very different from that obtained by mixing the sensations produced by the separate paints. The reason for this is that the color which would be transmitted by one pigment is absorbed in part by the other, and only that is transmitted which neither pigment absorbs.

A yellow disc and a blue one will give gray when blended by rotation, but yellow paint and blue paint mixed give green, for the blue absorbs red and transmits violet and green, while the yellow absorbs violet and transmits red and green. The two, therefore, absorb all but the green. The process may be expressed by an equation thus:

$$\begin{aligned}
(a)\quad \text{Yellow} &= \text{Red} + \text{Green} \\
(b)\quad \text{Blue} &= \text{Violet} + \text{Green} \\
(a)+(b)\quad \text{Yellow} + \text{Blue} &= \text{Red} + \text{Green} + \text{Violet} + \text{Green} \\
&= \text{White} + \text{Green}
\end{aligned}$$

The fact that green paint may be made by mixing blue and yellow paints led artists to reckon blue and yellow as primary colors, and green as a mixed color.

The classification suits the convenience of the artists, who continue to employ it.

Most of the greens employed by artists are, however, not made by mixing blue and yellow, but are simple pigments like the oxide of chromium, and many compounds of copper.

152. Colors by Interference. — Light waves, like water waves, sound waves, and all other waves, may interfere so as alternately to reënforce or destroy each other.

When light from any source reaches a given point by two paths which differ in length, it will in general interfere. If the light is of a single color, as red, the waves which have come by the two paths will destroy each other if the difference in length of the paths is a half wave length or any whole number of half wave lengths of red light. They will reënfore each other if the difference of path is a whole wave length or any whole number of wave lengths.

Suppose plane waves of wave length, l, to enter a dark box by two small openings, O, O' (Fig. 195), which are very close together, and fall upon a screen at N. The openings O, O' will act like sources of light, sending waves in every direction in the box.

If P be a point on the screen at such a distance from O that $O'P - OP = \frac{1}{2}\, l$, there will be a dark band at P where the waves meet in opposite phase and destroy each other.

If Q be a point such that $O'Q - OQ = l$, the waves will reënforce each other, and a bright band will be seen.

Had the light been white instead of being light of a single color, Q would have had a different position for each different wave length, and the band would have been spread out into a spectrum. The violet end would

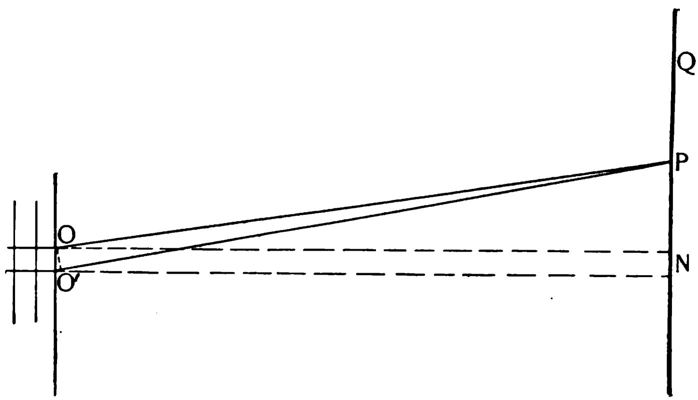


FIG. 195.

lie nearest the point N, the point where the normal to the wave before it entered the box meets the screen.

This bending of a wave when passing through small openings is called *diffraction.* A spectrum produced by diffraction is called a diffraction spectrum. In the diffraction spectrum the dispersion is proportional to the wave length, and the colors are arranged as indicated in Fig. 190, where the green is seen to occupy the

middle of the spectrum. In the refraction spectrum the violet end is dispersed much more than the red end (see Fig. 198).

A candle viewed through a slit in a card will show diffraction spectra. The candle should be several metres distant in a dark room and the card should be held near the eye. The slit may be made with the point of a sharp knife in the middle of a visiting card.

An umbrella held toward an arc lamp will show diffraction colors. The moon viewed through a window screen appears drawn out in the form of a cross.

153. Colors of Thin Plates. — When white light is reflected from two plane surfaces which are very near together the waves from one of the surfaces may interfere with those from the other surface, so as to destroy light of a certain wave length. The reflected light will show the color which is complementary to the one destroyed.

The colors seen on a soap bubble are produced in this way, as are also the gay colors of many insects and birds. The feathers of a peacock's tail would lose their brilliant colors if they were pressed flat enough to destroy the uniform arrangement of the little barbs of which they are composed. Mother-of-pearl ground to powder loses all its bright tints and appears like chalk dust.

154. The Spectroscope. — For a careful study of spectra an instrument called a *spectroscope* is used. It

consists of a stand carrying a circular plate, on which is mounted a tube called the collimator, *C* (Figs. 196 and 197), which has an adjustable slit at the end nearest the source of light, and a lens at the other end to make parallel the rays of light before they fall upon the prism, *P* (Fig. 196), or the grating, *G* (Fig. 197), which

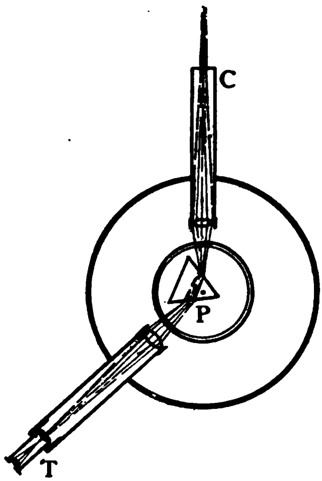


FIG. 196.

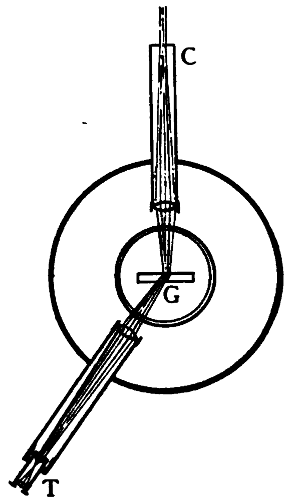


FIG. 197.

is to produce the spectrum. The spectrum is viewed by the telescope, *T*, which is mounted to point toward the centre of the circle.

Outside light may be cut off by a black cloth laid over the central part of the instrument. The instrument is usually graduated to degrees, so that the amount of deviation may be measured.

155. Spectrum Analysis. — The spectra shown by different bodies differ very much. They may, however, be divided into three classes:

I. *Band Spectrum.* — This is the spectrum shown by any white-hot solid or liquid. A gas flame shows such a spectrum, since the luminous part of the flame is composed of solid carbon particles. It is these particles of solid carbon that collect, as lampblack, upon any cold object held for an instant in the flame. The band spectrum shows all the colors of the rainbow, that is to say, it is produced by light of all wave lengths.

II. *Bright Line Spectrum.* — This is the spectrum of a glowing gas, like that obtained by burning salt in the flame of an alcohol lamp or Bunsen burner. It consists of one or more narrow lines which are always in the same relative position for a given substance, because in a gas the particles are free to vibrate as they please, and the atoms of each substance seem to have a definite period of vibration which gives rise to light of corresponding wave length, just as a given rod or string gives rise to sound waves of a particular pitch.

In solids the particles jostle against one another and so vibrate in all possible periods (Class I).

The bright line spectrum of a substance gives the chemist a means of detecting the presence in a compound of very minute portions of a substance. The process is called spectrum analysis.

III. *Dark Line or Band Spectrum.* — When white light is passed through a colored glass or solution or

vapor, some of the vibrations are absorbed, leaving the transmitted light wanting in certain wave lengths. The spectrum will appear crossed with black lines or bands. Sunlight and the light from many stars give spectra of this class.

156. The Solar Spectrum. — Fraunhofer was the first to explain the dark lines of the solar spectrum, which are now known as *Fraunhofer's lines.* He showed that a vapor will absorb waves of the same length as those which it emits. Thus sodium vapor placed between an arc light and a spectroscope will

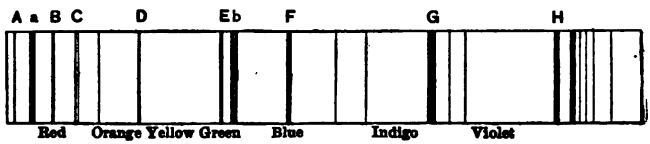


FIG. 198.

produce a dark line in the yellow in exactly the position where the yellow bright line appears when we view burning sodium.

The dark lines of the solar spectrum are, then, due to the presence of substances like sodium, iron and many others, in vapor form, in the atmosphere of the sun.

The principal Fraunhofer lines, designated by Fraunhofer with the letters of the alphabet, are indicated in Fig. 198.

157. Velocity of Light. — The speed with which light waves travel through space is so enormously great

that it is difficult for us to form any conception of it. For all ordinary purposes we treat it as being infinitely great. It was first measured by Roemer, a Danish astronomer, in 1675, by means of the difference in the apparent times of revolution of Jupiter's moons, depending upon whether the earth is moving toward or away from Jupiter. He found it took light about 1,000 seconds to cross the earth's orbit (186,000,000 miles), which gives for the velocity of light 186,000 miles per second.

A light wave would travel a distance equal to two or three times around the earth in the time it takes to wink. The light of the moon reaches us in a second and a half.

158. Intensity of Light. — If three square cards, *C*, *D*, *E* (Fig. 199), having heights of 10, 20, and 30 centimetres respectively, be placed at distances of 30, 60, and 90 centimetres from a source of light, *S*, it will be found that the shadow of *C* will exactly cover *D* or *E*. This is equivalent to saying that the light which fell on 100 square centimetres at a distance of 30 is distributed over 400 square centimetres at twice the distance and 900 square centimetres at three times the distance.

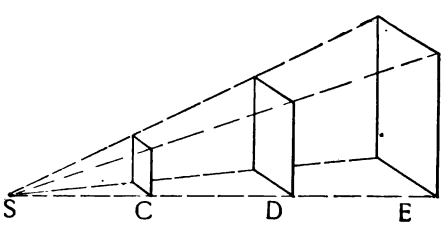


FIG. 199.

Light is thus seen to follow the same law of inverse squares that gravitation, magnetic attraction, and electric attraction follow.

The intensity of light is reckoned in candle power. To compare the intensity of two sources of light it is necessary to find a point at such a distance from both sources that the light received from both shall be equally intense. The distance of this point from each source is then measured and the intensity computed by the law of inverse squares.

Some methods of measuring candle power are explained in Part II.

159. Light and Electricity. — That the electric spark, the electric arc, and the electric current are sources of light is familiar to us all. There is a sense, however, in which a gas jet, a candle flame, nay, even the sun itself, is an electric light, for light waves are now known to be merely a particular class of electric waves of suitable wave length to impress the nerves of the retina.

The sun emits some waves which are much too long and others which are too short to affect the eye. Some of the long waves may be detected by means of the thermometer, some of the short ones affect the photographic plate.

The discovery of the identity of light waves and electrical waves grew out of the mathematical study by Maxwell of Faraday's researches in electricity.

More than twenty years after Maxwell published his electro-magnetic theory of light, Hertz, a young German physicist, undertook to find experimental proof of Maxwell's theory. After about eight years of patient research he found that the waves sent forth by the spark from an induction coil set up sympathetic vibrations in a coil of wire of suitable dimensions at some distance from the spark, and that the waves in his little secondary coil would produce a spark which gave a means of detecting and measuring the intensity of the waves at various distances from their source.

Hertz found that these long electrical waves (several metres in length) could be reflected, refracted, and diffracted exactly like light waves. In short, he demonstrated that Maxwell's theory is correct: that light is but one way in which the radiant energy of a luminous body may manifest itself.

We speak of solar light and solar heat, because having organs of sense to perceive light and heat we are made daily conscious of these particular manifestations of solar energy. We should rather think of *solar radiation* which may manifest itself to our senses as light or heat, which may produce chemical changes in the photographic plate or in tanning our skins brown or bleaching the color from fabrics, but which really consists of electrical waves different in wave length, but in all other respects alike.

160. Space Telegraphy. — Electrical waves have

found an important application in telegraphy at sea. Marconi has succeeded in telegraphing to considerable distances to and from vessels without the aid of wires. The messages are sent with as little difficulty during foggy weather as at any other time, a fact of great importance to sailors who are on a dangerous coast in bad weather.

There seems, at present, little likelihood that the system will replace wires on land, or cables for long distances across the ocean. Youd be surprised

161. New Forms of Radiation. — Lenard and Roentgen, following some lines of research suggested by Hertz, found that the radiations from the negative terminal of a vacuum tube (Crookes' tube) have the power of penetrating objects which are opaque to light waves and of causing a screen coated with fluorescent chemicals to glow brightly wherever the rays strike. These rays, both the Lenard rays and the X-rays of Roentgen, act upon a photographic plate when it is enclosed in a light-tight envelope.

The fact that the bones of our bodies are less transparent to the waves than is the flesh makes it possible to see a shadow picture of the bones of the hand when the hand is held between a luminous Crookes' tube and the fluorescent screen.

If a photographic plate take the place of the screen, we obtain a photograph of the bones of the hand, and of any foreign body which may be imbedded in the

flesh. Fig. 200 is reproduced from such a photograph. Bequerel has found that certain salts of uranium emit rays which have the same power of impressing the plate that the X-rays have.

FIG. 200.

The subject of electrical radiations has aroused great interest among physicists, and has already been made the subject of many investigations. Its chief applications hitherto have been in surgery, where it enables the surgeon to locate foreign bodies in the flesh without probing, and to know the exact nature and extent of fractures and abnormal growths of the bones.

Physicians are now able to examine the lungs and other internal organs with great ease, and without discomfort to the patient.

162. The Role of Wave Motion. — We are now

prepared to appreciate the important rôle which wave motion plays in our lives. By it we gain most of our knowledge of the world outside us, which comes to us through the eye and ear. Practically all of the energy which keeps our world alive and warm, the seat of ceaseless change, the home of myriad living plants and animals and men, instead of a dark, cold clod, comes to us from the sun, across the silent spaces, in waves of the intangible ether.

Exercises.

117. Why are the shadows cast by an arc lamp much sharper in outline than those cast by the sun?

118. What is the height of a tree which at a distance of 40 feet from the opening of a pinhole camera, gives an image 4 inches high on the ground glass of the camera 8 inches back of the opening?

119. When the sun is 45° above the horizon, how long is the shadow cast by a pole 90 feet high?

120. (*a*) Construct for the image of an object placed 8 cm. in front of a concave mirror of 24 cm. radius. (*b*) Is the image real or virtual, (*c*) upright or inverted, (*d*) enlarged or diminished?

121. Construct for the image of an object 8 cm. high placed 18 cm. in front of the mirror used in Ex. 120, and answer questions *b*, *c*, *d*, in regard to it.

122. Construct for the image of an object 8 cm. high placed 32 cm. in front of the same mirror, and answer the same questions in regard to it.

123. (*a*) Construct for the image of an object 12 cm. high placed 10 cm. in front of a convex mirror having a radius of

30 cm. (*b*) Is the image real or virtual, (*c*) upright or inverted, (*d*) enlarged or diminished?

124. Observe the image of your face in the convex side of a bright tablespoon, and explain the difference in the images depending upon whether the spoon is held vertically or horizontally:

125. What must be the height of a plane mirror in order that a person may see his whole length in it at once?

126. (*a*) Does an Indian spearing fish have to apply some knowledge of the laws of refraction? (*b*) Does he aim above or below the apparent position of the fish?

127. Where must the object be placed with reference to a convex lens if the image is to be (*a*) enlarged and real, (*b*) enlarged and virtual, (*c*) inverted and enlarged, (*d*) upright and enlarged, (*e*) real and of the same size as the object?

128. (*a*) In what respects do all images formed by concave (diverging) lenses agree? (*b*) In what respects may they differ?

129. To what are the colors due in (*a*) ice which has been shattered by a blow, (*b*) deep sea water, (*c*) shoal sea water, (*d*) rings about the moon, (*e*) the sunset sky?

130. Consult the index under "radiant energy," look up all the references and write a summary of all you have learned on that subject.

PART II.

LABORATORY EXERCISES.

PART II.—LABORATORY EXERCISES.

INTRODUCTION.

Physics is to-day the most exact of the sciences for the reason that the phenomena with which it deals are capable of accurate measurement. No one can form a just conception of the methods by which the great laws of physics have been developed who has not himself learned to make at least some simple measurements with a fair degree of accuracy. On the other hand, the student who has patiently and conscientiously performed a series of measurements with the highest degree of accuracy attainable by him with the instruments at his command has gained, not only manual and mental training, but moral training besides, for he has learned to prefer truth to falsehood and that exact statement of facts which leads ever to the discovery of truth, to the loose and ambiguous statement which tends to conceal the truth.

Measurement consists in obtaining an expression for the magnitude of the quantity to be measured in terms of some standard quantity called a unit.

The Fundamental Units. All physical quantities may be expressed directly or indirectly in terms of some one or more of the fundamental quantities, length, mass, time, the units of which are, respectively, the centimetre, the gram, and the second.

While the instruments by means of which the different quantities are measured may seem at first sight quite unlike each other, it is yet true that the actual observations made in measuring these quantities are very much alike. When the instrument has been adjusted for the observation, the observation itself will usually consist in reading or counting a number of linear divisions on a scale of equal parts and the estimation of a fraction of a division by means of the eye.

The student is urged, therefore, to observe in the performance of each exercise, not only the precautions there mentioned for securing accuracy, but, as far as they apply, the precautions observed in the preceding exercises. A little time spent in considering beforehand the probable sources of error in any problem may save time in the end.

The various sources of error to which observers are liable will be mentioned in the exercises. Accidental errors may be eliminated by making a number of independent observations and taking their mean, since it is probable that as many observations will give a result above the true value as below it. It is to be borne in mind that the mean of ten observations is, as a rule, entitled to ten times as much weight as is any one of the ten, even though that one be exactly the same as the mean. The student will understand without specific directions that every measurement which he is asked to make is to be repeated at least twice, often

ten or more times, and that all the observations are to be recorded, even if they should happen to be exactly alike.

If some observations differ much more from the mean than most of those taken, try to find a cause for the difference in the conditions surrounding the experiment, and, if possible, remove the cause of error.

The form in which the record of work is to be kept may be left to the taste of the instructor. All instructors will probably agree that the substance of the report should include (1) a statement of the object and the method of the exercise; (2) a description of the apparatus used, with such sketches as are needed to make the description clear; (3) an orderly record of the observations made; (4) a discussion of the results and method.

A Personal Hint. Not the least of the advantages to be derived from a course in laboratory work is the habit of order and neatness. We owe it not only to the people who are our laboratory mates, but to the people who are to be near us in after life, to see to it that the place which we have occupied is at least as clean and tidy when we leave it as when we came to it. If the teacher's directions in regard to the care of apparatus are explicitly followed by all the members of a class, the added pleasure derived from the work will alone more than repay the effort.

CHAPTER IX.

LENGTH.

163. Units of Length. — The metre is divided into 100 centimetres exactly as the dollar is divided into 100 cents. The centimetre is divided into 10 millimetres as the cent is divided into 10 mills. All lengths may be expressed in centimetres and decimals of a centimetre, and are understood to be so expressed in all computations unless the contrary is stated.

Small dimensions when expressed accurately are often mentioned in millimetres. Thus an inch is said to be equal to about two and a half centimetres, or more exactly 25.4 millimetres.

1. Table of Equivalents.

Length.

1 centimetre = .3937 in.
1 metre = 39.37 in. = 3.28 ft. = 1.0936 yd.
1 kilometre = 0.6214 mile.

1 inch = 2.54 cm.
1 foot = 0.3048 m.
1 yard = 0.9144 m.
1 mile = 1.6094 km.

Area.

1 sq. cm. = 0.155 sq. in.
1 sq. m. = 10.714 sq. ft. = 1.196 sq. yd.

1 hectare = 2.471 acres.
1 sq. km. = 0.386 sq. mile.

1 sq. in. = 6.452 sq. cm.
1 sq. ft. = 929.034 sq. cm.
1 sq. yd. = 0.83613 sq. m.
1 acre = 0.4047 hectare.
1 sq. m. = 2.59 sq. km.

Volume.

1 cu. cm. = 0.061 cu. in.
1 cu. m., or stere = 35.315 cu. ft.
1 litre = 1.0567 qt. (U. S.).

1 cu. in. = 16.387 cu. cm.
1 cu. ft. = 0.028 cu. m.
1 cu. yd. = 0.765 cu. m.

164. Estimation of Tenths. — Since the smallest division of the common metre stick is the millimetre, we can measure with it directly to millimetres. We should always try, however, to estimate fractions of a millimetre. We may at first find it more natural to estimate fourths; but since it is frequently possible to estimate closer than fourths, it is better to form the habit at once of estimating tenths. If the length is a certain number of millimetres plus a little more than $\frac{1}{2}$ call the fraction .6, if a little less than $\frac{3}{4}$ we may call it .7, and so forth.

EXERCISE 131. — **To measure the length and breadth of a laboratory table.**

Apparatus. — Metre stick, a square block of wood or iron, try-square.

Directions. — To measure the width of the table,

place the square as shown in Fig. 201, at the left, with the ruler resting firmly against it. The ruler is then at right angles to the side of the table, and the end of the ruler is exactly over the edge of the table. Let your assistant hold the ruler in place while you remove the square, place it as shown at the right-hand side of Fig. 201, and read on the metre scale the centimetres, millimetres, and tenths of a millimetre at the point exactly opposite the edge of the square. Make such measurements at intervals along the length of the table.

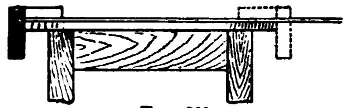


FIG. 201.

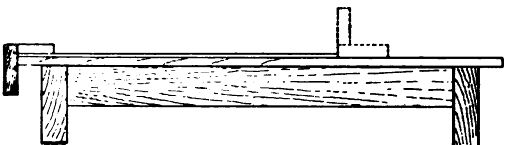


FIG. 202.

To measure the length, place the metre stick against the square as in Fig. 202. Since the table is probably more than a metre long, place the square against the other end of the stick. Hold the square in place while you remove the stick, and put in its place against the square, a square block. Now hold the block in place, remove the square, and measure from the block (see Fig. 203).

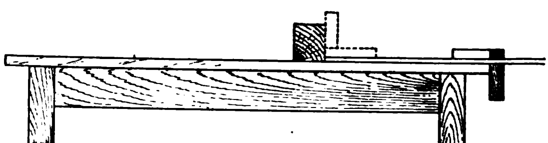


FIG. 203.

Do not mark the table, as this will not only disfigure the table, but prejudice the next person who measures it. This remark holds good in general: no marks are to be made upon apparatus except under the teacher's directions.

Record your results without reporting them to your assistant, then let him make a similar series of measurements while you assist. After you are both through compare results. If either has made a serious blunder, the comparison will show it.

It is well to record both your own results and those of your assistant, keeping them apart, however, and designating them by name.

Note the suggestions in regard to the record of results on page 267. The measurements may be tabulated in some such form as the following:

DIMENSIONS OF TABLE NO. 4.

First breadth at East end. First length at North side.

Breadth in Cm.

Obs. No.	Breadth.	Averages.		Observer.
1	86.27			Self.
2	86.28			,,
3	86.29			,,
4	86.29			,,
5	86.30	86.286		,,
6	86.26			Stephens.
7	86.28			,,
8	86.30			,,
9	86.31			,,
10	86.31	86.292	86.289	,,

The differences in your own results will indicate differences in the dimensions of the table at different points, provided that your assistant's results show corresponding differences.

Length in Cm.

Obs. No.	Length.	Averages.		Observer.
1	186.26			Self.
2	186.27			"
3	186.26	186.263		"
4	186.25			Stephens.
5	186.26			"
6	186.26	186.257	186.260	"

EXERCISE 132. — **To measure the height of a column of liquid in a glass tube.**

Apparatus. — Metre stick, two blocks, flat stick or strip of plate glass.

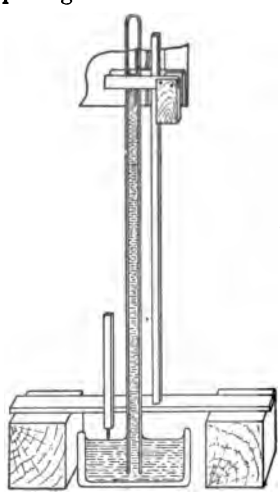


FIG. 204.

Directions. — Place a flat stick in the horizontal position across two blocks as near the tube as possible, and measure from the top of the stick (Fig. 204) to the upper liquid surface, and then from the same level to the lower liquid surface. The sum (or difference) of these two measures is the height required.

Care must be taken in making measurements (*a*) to define the surface: that is considered to be the lowest part of the surface of a liquid which, like water, wets

the glass, the highest part of a mercury surface; (*b*) to see that the metre stick is held in the vertical position; (*c*) to make sure that the eye is in the same horizontal plane with the surface to be measured. A good way to do this is to hold a bit of mirror or a piece of bright tin against the back side of the metre stick and slide a try-square along the front of the metre stick till the edge of the square, its reflection in the mirror, and the surface are all in line.

The error due to the eye's being held in any position not in a perpendicular line to the scale at the point of observation is called *parallax.* It will be necessary to guard against parallax in almost every measurement you will ever make. Parallax is greater the farther the object is away from the scale.

After making two careful measurements change the height of your base line, by turning the blocks or using a stick of different thickness, and make two more.

To measure downward from the block to the lower surface a piece of metric ruler having a point projecting from the end should be used. For acids the point should be of glass. If the ruler is sawed off an amount equal to the length of the point, the reading on the ruler will be the length measured. Otherwise the distance from the point to some point on the ruler must be measured by holding the piece of ruler beside the metre stick while the point and the end of the metre stick rest upon a piece of glass or other hard smooth surface.

EXERCISE 133. — **To measure the distance between two points with the diagonal scale.**

Apparatus.—Dividers, diagonal scale. For the dividers, any dividers or compasses with sharp points will answer.

The diagonal scale is shown in Fig. 205. It is designed to aid in measuring short distances more accurately than can be done with the simple scale by means of a device for reading tenths. The scale is ruled in centimetres toward the left from zero. To the right of zero is a single centimetre space divided into tenths by diagonal lines, the bottom of each line

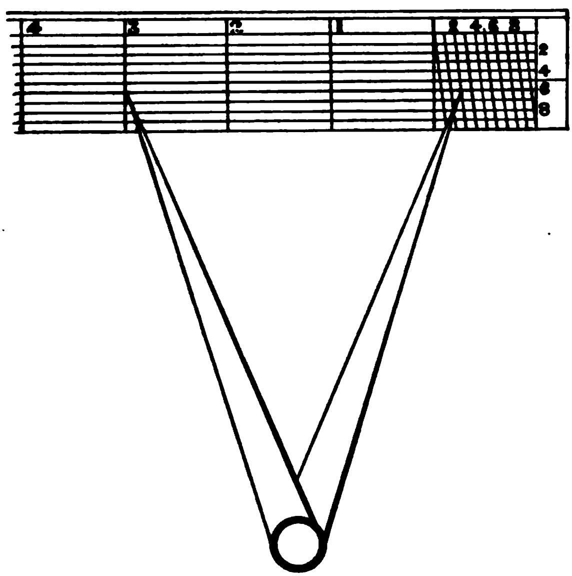


FIG. 205.

being just 1 millimetre to the right of the top end of the line. The space between the top and bottom of the scale is divided by horizontal lines into ten equal parts.

The first horizontal line must intersect the first, or zero, diagonal line .1 millimetre to the right of the nearest straight line, which is the zero of the scale. The intersection of horizontal 5 with diagonal 4 is 4.5 mm. to the right of zero of the scale.

Directions. — Adjust the dividers so that the hinge does not work too easily and spread the points so that when one is placed upon the centre of one of the given points the other will exactly reach to the centre of the second point. Now place the dividers upon the scale with the left leg upon that centimetre division which will bring the right leg within the diagonal square. Starting with the dividers at the top of the scale move downward, keeping the left leg on the vertical line and both legs always in the same horizontal line till the right leg meets the intersection of a diagonal line with a horizontal. The number of the centimetre line will give the centimetres, the number of the diagonal line the millimetres, and the number of the horizontal line the tenths of a millimetre. The reading in Fig. 205 is 3.26 cm.

CAUTION. — *Do not press upon the scale with the dividers, else it will soon become blurred and useless.*

After making a measurement, close the dividers and repeat the measurement.

EXERCISE 134. — **To test your ability to estimate tenths of a scale division.**

Apparatus. — Dividers, diagonal scale, metre stick.

Directions. — Make three measurements of the distance between two points not before measured, using

the dividers and metre stick, and estimating tenths of a millimetre as well as you can with the eye, then make three measurements, using the diagonal scale, and compare your results. Try not to let any of your measurements be influenced by those already made. If any of a set of measurements is better than the rest it ought to be the last one rather than any of the others.

EXERCISE 135. — **To measure the diameter of a sphere (a) with the calipers, (b) with the metre stick.**

Apparatus. — Outside calipers, two rectangular blocks, flat stick, metre stick, heavy hammer handle.

The calipers shown in Fig. 206 are called outside calipers, and are used in measuring the diameters of spheres and cylinders.

Directions. — (*a*) Indicate different diameters on the sphere with chalk. Open the calipers a little wider than what you judge to be the diameter of the ball and make a trial. If they are open a little too wide take the calipers loosely in the hand with the hinge between the thumb and forefinger, and tap gently with one leg of the calipers upon the hammer handle or any block that will neither mar the calipers nor be damaged by them. A little practice will enable you to judge how hard a blow to strike. If the legs are a little too close together slip the hammer handle between them and give a slight blow upon it with the inside of the caliper leg.

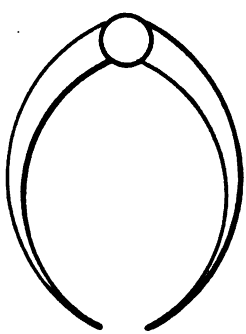


FIG. 206.

When the sphere will exactly slip between the points, measure the distance the two points are apart with the metre stick or diagonal scale, open the calipers and then measure another diameter. Measure six diameters.

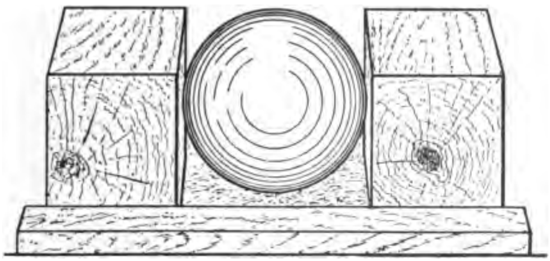


FIG. 207.

(*b*) Place the blocks upon the table against the straight stick and notice whether they fit squarely against each other (see Fig. 207). If all sides are not equally square choose the best sides, place the sphere between the blocks and press the blocks against it. See that the blocks are pressed firmly against the stick.

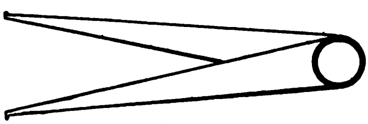


FIG. 208.

Now measure the distance between the blocks on both sides, near the top and near the bottom. Repeat the measurement for three diameters at right angles to each other.

EXERCISE 136. — **To measure the inside and outside diameters of a tube with the calipers.**

Apparatus. — Outside calipers, inside calipers (Fig. 208), diagonal scale.

Directions. — Proceed as in Exercise 135 (*a*), making four measurements at each end for the inside diameter, two near the end at right angles to each other, two as far in as the calipers will go.

Make eight measurements of the outside diameter along one element and eight more along the element at right angles to the first. Record the first eight results in one column and the second eight in another, and compare the values to see if there is any indication that the tube is flattened. Glass tubes are not unlikely to be thus slightly elliptical in cross section.

With the outside calipers measure the thickness of the walls of the tube, and see how it compares with the thickness obtained by taking half the difference between the outer and inner diameters of the tube.

165. The Vernier. — The vernier is a device for estimating tenths of a scale division so simple in principle and construction that it may be applied to a great variety of instruments.

The vernier consists of a movable scale arranged to slide along a fixed scale. For simplicity let us describe a vernier which reads tenths. When the zero of the vernier (Fig. 209) is opposite the zero of the scale the tenth division of the vernier will be found to be opposite the ninth division of the scale. Each division of the vernier is just .9 of a scale division in length. If then the vernier be set so that 1 of the vernier is opposite 1 of the scale, 0 of the vernier is .9 to the

left of 1 or .1 to the right of 0. But since the zero of the vernier is the point to be read, the reading is .1 scale division. The reading on the scale shown in Fig. 209 is 12.3.

For the 0 of the vernier is past 12 of the scale and 3 of the vernier is opposite 15 of the scale, so that the 0 of the vernier is $3 \times .9 = 2.7$ to the left of 15, or the reading is $15 - 2.7$.

In practice we need not notice what division of the scale the vernier is opposite. We do observe what division of the scale the zero of the vernier is past and

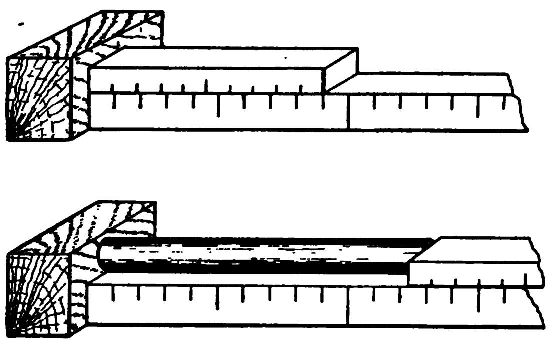


FIG. 209.

what division of the vernier coincides with a division of the scale.

Verniers are often made to read twentieths of a millimetre, fiftieths of an inch, sixtieths of a degree, etc.

Before using a vernier set it so that its zero coincides with a division of the scale, follow along the vernier till the next division of the vernier which coincides

with a division of the scale is found, and count the divisions on the vernier from the first coincident line to the second. If the number is n divisions the vernier reads $n$ths of a scale division.

Exercise 137. — **To measure the length of an object with the simple vernier.**

Apparatus. — Metre stick and vernier, block.

Directions. — Place the object and metre stick side by side against the block and slide the vernier along till it touches the end of the object (see Fig. 209). Now read the vernier as already directed.

Exercise 138. — **To measure the diameter and length of a small cylinder with the vernier gauge.**

Apparatus. — Vernier gauge.

The vernier gauge (see Fig. 210) consists of a steel scale provided with two jaws at right angles to its length. One of

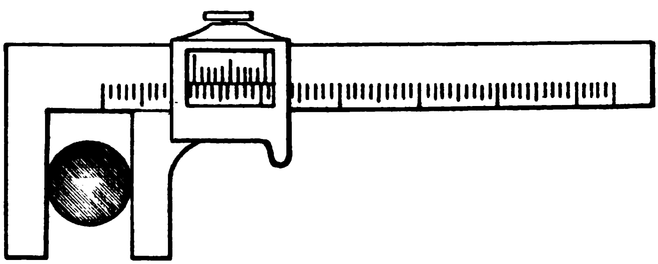


Fig. 210.

the jaws is fixed to the scale; the other, to which a vernier is attached, slides along the scale. When the jaws are shut the zero of the vernier should coincide with the zero of the scale, for the distance apart of the jaws is then zero. The movable jaw is usually provided with a screw which holds the jaw in place while the reading is being taken.

Directions.—Place the object between the jaws of the gauge, press the movable jaw gently but firmly against it, set the screw, and read the scale and vernier. Make ten measurements of the diameter and four of the length.

Correction. — As remarked above, the reading should be zero when the jaws are closed, but it may happen that the jaws have been slightly bent, so that the reading will vary one or two tenths from zero. Close the jaws and read the vernier. The error, if any, should be recorded as the error of zero. It is evident that if the instrument reads 0.1 at 0 it will read too high by .1 at other points as well, and .1 must be deducted from all readings, or what amounts to the same, from the average of the readings taken. Bear in mind that the correction is —.1 when the error is +.1, or in general the correction is the error with the sign changed, or the correction is the amount to be added to make the result correct.

EXERCISE 139. — **To measure the internal diameter of a ring or hollow cylinder with the vernier gauge.**

Apparatus. — Two vernier gauges.

Directions. — Close the jaws of the first gauge and measure with the second gauge the distance across the jaws. This amount must be added to all inside readings taken with this particular instrument, since it is a negative error.

Record it thus:

Correction for width of jaws of vernier gauge No. 4,	1.60 cm.
Correction for error of zero,	– 0.01
Total correction for inside measurement,	1.59 cm.

The gauge used for inside measurements of tubes or rings must have its jaws rounded on the outside. The measurements are to be taken as in Exercise 138.

166. Eye Estimation of Lengths.—It is often useful to be able to form a rough estimation of distances by means of the eye alone, as, for instance, if we are selecting a block to support an object at a certain height. One who is much accustomed to such eye estimation will detect any gross error of measurement by its means. Now that the student has become familiar with the unit of length he will find it useful exercise to estimate distances habitually before measuring them.

EXERCISE 140.—**To estimate and measure the length of several objects.**

Apparatus.—Metre stick.

Directions.—(*a*) Bend the forefinger and estimate the length from the tip of the nail to the first joint. Record your estimate and then measure the length. (*b*) Spread the hand and estimate the distance you can span with the thumb and middle finger. Record and measure. (*c*) Estimate and measure the length of your foot, allowing, as well as you can, for the distance the shoe projects at each end. (*d*) Estimate and measure the distance from the tip of your middle finger to your elbow. (*e*) Estimate and measure the length of your steps by taking one fifth of the distance covered in five steps. (*f*) Estimate and measure the height of another student.

In olden times these lengths, when referred to the person of the king, served as units of length and corresponded to (*a*) the inch, (*b*) the span, (*c*) the foot, (*d*) the cubit, (*e*) the yard, (*f*) the fathom.

The form of an object and its position influence our judgment of its dimensions.

EXERCISE 141. — **Estimation in different positions.**

Directions. — (*a*) Estimate and measure the width and height of the two parts of the eight shown in Fig. 211.

(*b*) Estimate and measure the length of a cylinder lying on its side, and of another cylinder standing on its end. A barrel is a good object for the purpose.

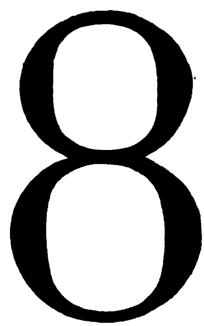


FIG. 211.

The objects you have measured will, if their dimensions are borne in mind, aid you in making other estimates of length.

167. The Micrometer Screw. — A still more accurate instrument than the vernier for measuring small dimensions is the micrometer screw. It consists of an accurately cut screw, the head of which is divided into a number of equal parts. Its simplest forms are the micrometer gauge and the spherometer.*

EXERCISE 142. — **To measure the diameter of a wire with the micrometer gauge.**

Apparatus. — Micrometer gauge.

* More complicated forms are the filar micrometer, used in microscopes and telescopes, and the dividing engine, used in ruling accurate scales.

The micrometer gauge (see Fig. 212) consists of a bent arm, one end of which is threaded to receive a screw, the other end containing an adjustable stop, *S*. The linear scale on the shank shows how many millimetres the screw is from the stop. If the distance between the threads of the screw be ½ mm. the screw will advance 1 mm. for every two turns. The head is divided into equal parts. If the threads are ½ mm. apart the head will be divided into 50 parts. If the threads are 1 mm. apart the head will be divided into 100 equal parts. In either case one division on the head corresponds to a forward movement of .01 mm. by the screw. By estimating tenths of a division we may read .001, but we are to understand that our result cannot be relied upon as correct to .001, since the error in setting the screw is usually .002 or more.

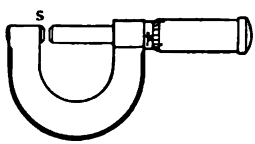


FIG. 212.

Directions. — Bring the screw and stop in contact with a gentle pressure. A uniform pressure may be secured by grasping the head of the screw loosely in the fingers and turning till the fingers slip. Note and record the zero error. Place the wire against the stop and bring the screw up to it with about the same pressure used in determining zero.

If the threads are ½ mm. be careful to notice whether the reading on the scale is a whole number of millimetres or a whole number plus a half. The reading on the instrument shown in Fig. 212 is 2.068 mm. If the screw were turned one revolution backward it would be 2.568 mm.

Take ten readings of the diameter of the wire, average the results, and determine the gauge number from Table 2. Determine the gauge number also by means of an American wire gauge such as is shown in Fig. 213.

2. VALUE IN MILLIMETRES OF BROWN & SHARP WIRE GAUGE NO.'S.

No.	*mm.*	*No.*	*mm.*	*No.*	*mm.*	*No.*	*mm.*
1	7.348	9	2.906	17	1.150	25	0.455
2	6.544	10	2.582	18	1.024	26	0.405
3	5.827	11	2.305	19	0.912	27	0.361
4	5.189	12	2.053	20	0.812	28	0.321
5	4.621	13	1.828	21	0.723	29	0.286
6	4.115	14	1.628	22	0.644	30	0.255
7	3.656	15	1.459	23	0.573	31	0.227
8	3.264	16	1.291	24	0.511	32	0.202

168. The Spherometer. — The spherometer is a micrometer screw of a form especially adapted to the measurement of the curvature of spherical surfaces. It may also be used for measuring the thickness of small glass plates. It consists of a tripod, the three pointed legs of which form an equilateral triangle (Fig. 214). Through the centre of the tripod passes the pointed screw, at the top of which the

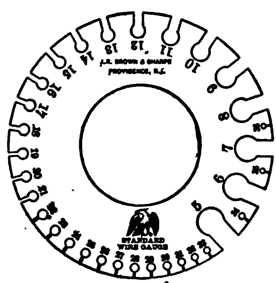


FIG. 213.

head is enlarged to form a disk which is graduated on its edge. The whole turns of the screw are read from the vertical scale, *S*, the fractions of a turn from the disk.

EXERCISE 143. — **To measure the thickness of a small glass or mica plate with the spherometer.**

Apparatus. — Spherometer, glass plate.

Directions. — Place the instrument upon a piece of plate glass and turn the screw till it touches the glass. If it is turned a little too far the tripod will be lifted so as to wobble on its points. Turn the screw back till the wobbling motion ceases. All four points are now in one plane and the reading is the zero reading. If the scale reads from the bottom up, the zero will be near the middle of the scale to permit of readings being made upon concave surfaces. Take five readings of the zero in different positions upon the large plate glass. Having determined the zero, raise the screw till the plate to be measured will slip under it, bring the screw to contact and read the instrument. Repeat four times on different parts of the plate. The average of these five readings less the average zero reading is the thickness of the plate.

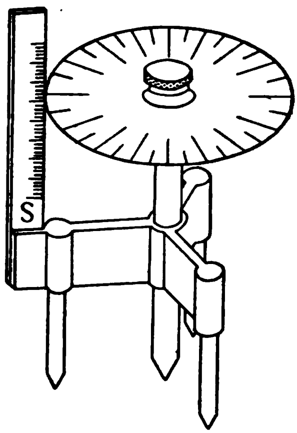


FIG. 214.

EXERCISE 144. — **To measure the radius of a large lens or other spherical surface.**

Apparatus. — Spherometer and glass plate, dividers and diagonal scale.

Directions. — Determine the zero reading as in Exercise 143. Raise the screw, place the spherometer upon the lens, bring the screw to contact and take a reading.

Repeat at five different places as far apart as possible upon the surface of the lens. The difference between the average of these readings and zero is the height (h, Fig. 215) of the point p above the plane which passes through the points of the tripod. It remains to measure d, the distance of p from each of the three points of the tripod when all the points are in the same plane. To measure d place the instrument

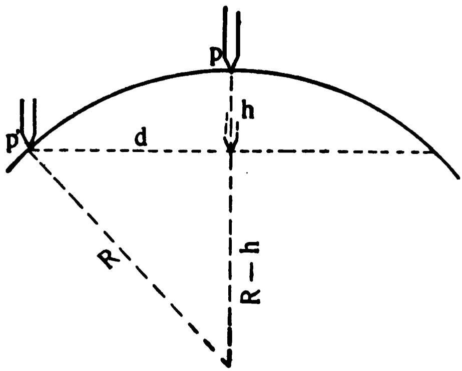


FIG. 215.

upon a flat page of your notebook and press the points down until they make dents in the paper. Bring the screw down till it also makes a mark. Measure the distance of the central dot from each of the others with the dividers and scale.

If R be the radius of the sphere of which our lens is a segment it is evident from geometry that

$$(41) \quad R^2 = (R - h)^2 + d^2 = R^2 - 2\,Rh + h^2 + d^2$$

whence:

$$2\,Rh = h^2 + d^2$$

$$(42) \quad R = \frac{h^2 + d^2}{2\,h}$$

from which equation, by substituting the values of d and h obtained by measurement, we obtain R.

CHAPTER X.

MASS.

169. Mass Defined. Units of Mass. — By the mass of a body we mean the amount of matter which composes it. Since gravity at any place on the earth acts with equal force upon equal masses, we require for the measurement of an unknown mass (1) a standard mass, (2) an instrument for comparing the force exerted by gravity on the standard mass and the unknown mass. The standard mass in the metric system is the gram, which is defined as the mass of a cubic centimetre of water at its temperature of greatest density (4 degrees Centigrade). For the measurement of small masses sets of weights are made, usually of brass, containing masses of 1 gram, 2 grams (two), 5 grams, 10 grams, 20 grams (two), 50 grams, 100 grams, 200 grams (two), 500 grams. For coarse weighing weights of 1,000 grams, called kilograms, are made. For fractions of a gram sets of centigram weights, usually made of sheet platinum, and for very delicate weighing milligram weights, often made of sheet aluminum, are used.

3. Equivalents in Weight.

1 gram = 15.4324 grains.

1 kilogram = 2.2046 pounds.

1 grain = .0648 grams.
1 ounce (av.) = 28.35 grams.
1 ,, (tr.) = 31.1 grams.

170. The Spring Balance. — If a force be exerted to elongate a spiral spring the elongation will be proportional to the force. This principle is made use of in the spring balance.

The spring balance in common use for coarse weighing has been graduated by the maker so that we have only to suspend it by the ring at the top, hang an unknown mass from the hook attached to the bottom of the spring, and read off the weight directly from the scale (see Fig. 216). The difference in the force of gravity at different places is too small to be detected by such a balance and may therefore be disregarded.

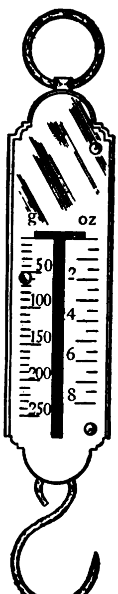


FIG. 216.

A very delicate form of spring balance is that known as Jolly's balance, shown in Fig. 217. It consists of a long spiral spring made of fine wire, suspended in front of a scale. The elongation produced by suspending a known weight from the wire is read from the scale. From a series of such determinations the average elongation for one gram is determined.

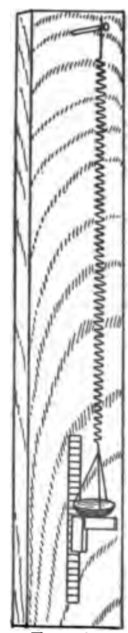


FIG. 217.

EXERCISE 145. — **To weigh a small object with Jolly's balance.**

Apparatus. — Jolly's balance and some small weights.

Directions. — Choose a well-defined point near the bottom of the spring and read its position on the scale (an index is usually attached). To avoid parallax the scale is usually ruled on a mirror. When the index covers its image in the mirror read the position of the index on the scale. If a piece of metre stick is used for a scale, as in the home-made form of balance shown in Fig. 217, the reading is taken from the top of a try square which just touches the pan. Record the zero reading, place the object to be weighed in the pan and read again. Repeat both observations after shifting the zero by adding a small additional weight to the pan. Now find the value of one gram in scale divisions by placing in the pan that one of the small weights which will give a reading nearest that given by the unknown weight. If five grams extended the spring 12.5 cm., one gram would extend it 2.5 cm. An object which extends the spring 16.3 cm. would weigh 6.52 grams.

171. The Lever Balance. — The principle of moments furnishes a convenient method of comparing masses. If a rigid bar (Fig. 218) is supported from

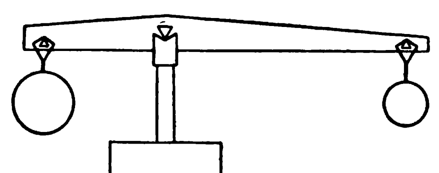


FIG. 218.

a triangular fulcrum a little higher than its centre of

gravity, it will come to rest in a horizontal position when the sum of the moments of the forces on the left-hand side is equal to the sum of the moments on the right-hand side. If the right arm is made much longer than the left arm, while the left arm is made enough heavier so that the lever balances, it is evident that a single

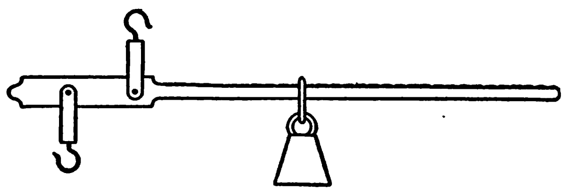


FIG. 219.

weight may be made to balance different masses by varying the position of the weight on the right arm.

Such balances, now known as steelyards, were in use by the ancient Romans. The long arm was graduated to equal parts by notches into which a link supporting

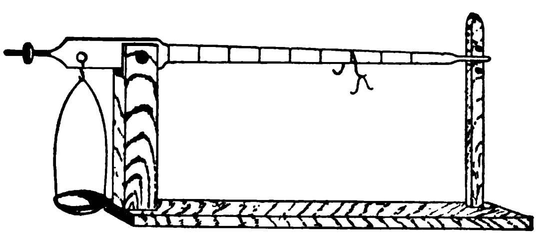


FIG. 220.

the weight could be dropped. From the short arm was suspended a hook for supporting the object to be weighed (see Fig. 219). A form adapted to delicate weighing is shown in Fig. 220.

It is obvious that when the two weights are at equal distances from the fulcrum the weights are equal when the beam comes to rest in a horizontal position. When the long arm is n times as long as the short arm the weight on the short arm is n times as great as the weight on the long arm.

EXERCISE 146. — **To weigh an object with a steelyard.**

Apparatus. — Common steelyard.

Directions. — Support the steelyard in the left hand by the hook farthest from the heavy end if there are but two hooks, from the second if there are three. Hang the body to be weighed from the hook near the heavy end and move the bob till a notch is found in which the lever will be horizontal. If the object is too heavy turn the steelyard over and use the other scale. After reading the scale and recording the weight, weigh a known weight and then measure the distances between the knife edges and the length of a scale division, and compute, by the law of moments, the weight of the bob.

EXERCISE 147. — **To weigh an object with a rider balance.**

Apparatus. — Balance with unequal arms, set of rider weights.

Directions. — Test the zero of the instrument and if necessary adjust it by means of the screw. Place the object in the pan and adjust one of the larger riders till it will a little less than balance the object and add smaller riders till the beam is in equilibrium. The sum of the weights balanced by the different riders is the

weight of the object. If in doubt about the weight of the different riders weigh a known weight, as a five-cent nickel, which, when new, weighs five grams.

172. The Lever Balance with Equal Arms. — The hand balance (Fig. 221) with equal arms is probably the oldest balance known. It was in universal use for weighing money before the practice of government coinage became common. The triangular pin passes through the centre of the beam and rests in a double link which is held in the hand or fastened by a cord to some convenient support. Near the ends of the beam and at equal distances from the centre are suspended from small pins the pans of horn or metal which hold the weights. Such balances are in common use to-day by university students in the German chemical laboratories and are the best cheap balances to be had.

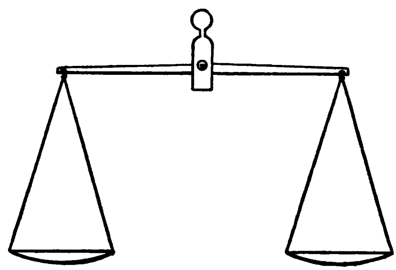


FIG. 221.

A more convenient form of the lever balance is the prescription balance or beam balance shown in Fig. 222. It is identical in principle with the hand balance, but the beam is supported on the top of a pillar and a long and light pointer indicates on the scale small variations of the beam from the horizontal position. The pillar

may be made in two parts, the upper being arranged to slide within the lower. In the lowest position of the beam the pans rest upon the base. By pressing a lever the beam is lifted and the pans are set free.

EXERCISE 148. — **To weigh an object with the common balance.**

Apparatus. — Hand or beam balance, set of weights.

Directions. — Allow the beam to swing and find where the pointer comes to rest. If this point is near

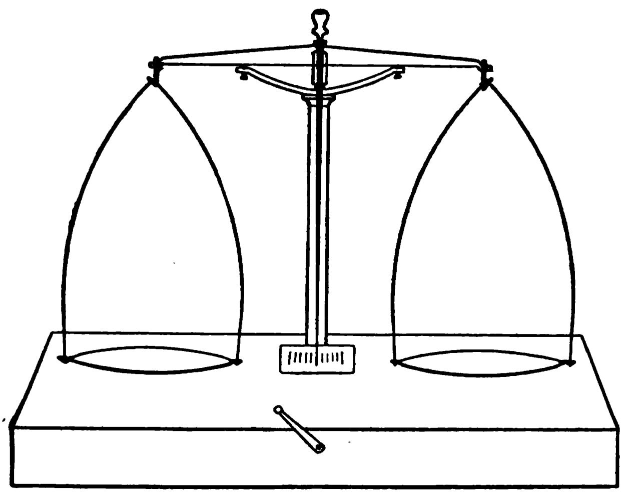


FIG. 222.

the centre of the scale, note the exact point; if it is not near the centre, add enough fine shot, grains of sand, or bits of paper to the proper pan to bring it near the centre. Arrest the beam by means of the lever, *l*, or in the hand balance by holding the beam with the left

hand. Place the object in the left pan and put such weights in the right pan as you judge will about balance the object. Release the beam a little to see which side is heavier. Never allow one end of the beam to go very much lower than the other, especially when a load is on the balance, as the knife edge of the fulcrum is likely to be injured thereby. A good rule is: *Never let the pointer swing off the scale.*

When the weights are found which will bring the pointer nearly to zero let the beam come to rest and note the point of rest. Suppose that there are in the pan 4.64 grams and that the point of rest is 4 divisions to the right of zero. You now add 1 cg. and find the point of rest 2 divisions to the left. One cg. has moved the pointer 6 divisions. It would have taken 4/6 cg. to move it 4 divisions, that is, to bring it to zero. The exact weight is therefore 4.647 grams. Support the beam and remove the objects and weights, counting the weights again to make sure you have made no mistake.

EXERCISE 149. — **To test a balance by double weighing.**

Apparatus. — Balance, weights.

Directions. — We have assumed in Exercise 218 that the arms of the balance are exactly equal in length. This is by no means certain to be the case, especially in a cheap balance, nor is it necessary that the arms be exactly equal in order that we may weigh correctly, provided we know the ratio of the lengths.

Weigh an object as accurately as possible in the left-hand pan. Then weigh the object in the right-hand

pan. If we call the weight required to balance the object when it is in the left pan W and the weight required to balance it when in the right W', while its true weight is M, it is evident from the principle of moments that if R and L be the length of the right and left arms of the balance (see Fig. 223),

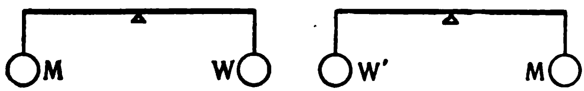


FIG. 223.

$$(a) \qquad LM = RW$$

$$(b) \qquad RM = LW'$$

Dividing (b) by (a)

$$(c) \qquad R/L = LW'/RW$$

Multiplying both sides by R/L

$$(d) \qquad R^2/L^2 = W'/W$$

whence

$$(43) \qquad R/L = \sqrt{W'/W}$$

But from (a), by dividing by L,

$$(45) \qquad M = \frac{R}{L} W$$

It follows that to find the true weight with a balance whose arms are not equal we have only to determine R/L by double weighing once for all and afterward multiply the apparent weight of the object in the left-hand pan by R/L.

It is further true that for nearly equal arms

$$(46) \qquad M = \frac{W + W'}{2}$$

Having found the true weight by double weighing, check your result by the method of counterpoising described in the following exercise.

EXERCISE 150. — **To weigh by the method of counterpoising.**

Apparatus. — Balance and weights, dish of sand.

Directions. — With the object in the left pan put enough sand or shot in the right pan to exactly balance it. Now remove the object and put weights enough in its place to balance the sand.

EXERCISE 151. — **To weigh an object by the method of swings.**

Apparatus. — Balance and weights.

Directions. — Find the zero of the balance. In a delicate balance the beam does not come to rest for some time, but the point of rest may be determined by observing the extreme position of the pointer for three successive swings. Since the amplitude of swing is gradually growing less, owing to the resistance of the air, the average of the first and second swings would give us a point a little to the right of zero, if the first swing was toward the right, while the average of the second and third would give us a point a little to the left of zero. Thus in the example illustrated in Fig. 224 the swings were *

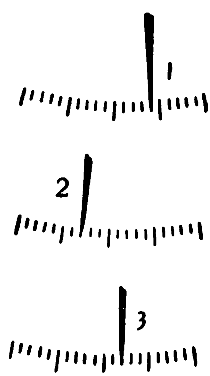


FIG. 224.

* To avoid negative readings the scale is numbered from left to right usually in twenty divisions, thus bringing the zero position at the 10th division.

$$\begin{array}{l} 1 \ldots\ldots\ldots\ldots\ldots 14 \\ 2 \ldots\ldots\ldots 7 \\ 3 \ldots\ldots\ldots\ldots\ldots 12 \end{array} \qquad \left.\begin{array}{l} \dfrac{14+7}{2} = 10.5 \\[2ex] \dfrac{7+12}{2} = \;\; 9.5 \end{array}\right\} = 10$$

Hence to find the zero, average the right-hand readings and the left-hand readings separately and take the mean of the two averages.

$$\text{Thus:} \quad \frac{7 + \dfrac{14+12}{2}}{2} = 10$$

After finding the point of rest arrest the beam, release it again and take a new set of readings. If the two sets give results agreeing to a tenth of a division they may be averaged and the mean taken as the zero. Otherwise a number of trials must be made. If the balance is in a strong draft currents of air may penetrate the case. One object of repeating observations several times is to find out whether our results really possess the degree of accuracy of which the instruments and methods used are capable.

For simplicity the readings given in the example were all whole numbers. In practice tenths of a division should always be estimated. If the zero falls more than one scale division from the centre the instructor should be asked to adjust the balance.

Having determined the zero, which we will call p_0, place the object in the left pan and balance it with

weights, W, as in Exercise 220. When the adjustment has been made by means of the rider to within 1 mg. find the point of rest p_1, by swings, add 1 mg. by means of the rider, and find another point of rest p_2. The weight of the object is then

$$(47) \quad M = W + \frac{p_1 - p_0}{p_1 - p_2}$$

173. Density.—By the density of a substance is meant the mass of unit volume of that substance. For purposes of comparison it is common to speak of relative densities, taking as our standard of comparison the weight of unit volume of water at its temperature of greatest density (4 degrees C.).

If a certain volume of a given substance is found to weigh 7.7 times as much as an equal volume of water, the relative density or specific gravity of the substance is 7.7.

One cubic centimetre of water at 4 degrees C. weighs one gram, hence the number expressing the weight of 1 cu. cm. of a substance expresses also its relative density or specific gravity.*

If the body has a volume of v cu. cm. we must divide its mass m by v to find the mass of 1 cu. cm.; hence, in general, to find the density of a body we divide its mass by its volume, and our fundamental formula for density is

$$(48) \quad d = m/v$$

* For the sake of brevity we shall use the word *density* to denote relative density.

The mass m we usually find by weighing the body. The method of finding the volume must be adapted to the case in hand. Density as just defined should, strictly speaking, be distinguished as *volume density*. The mass of unit length of a string or wire, for example, is called its *linear density*, while the mass of unit surface of a sheet of uniform thickness is called its *surface density*. When nothing is specified to the contrary, volume density is always understood.

EXERCISE 152. — **To find (a) the linear density and (b) the volume density of a wire.**

Apparatus. — Metre stick, micrometer gauge, balance and weights.

Directions. — (*a*) Measure the length of the wire with the metre stick and its weight with the balance.

$$(49)\quad \textit{Linear density} = \frac{\textit{mass}}{\textit{length}}$$

(*b*) Measure the diameter of the wire and compute its volume from the formula $v = 3.1416\, r^2 l$, where v is the volume, r the radius, and l the length of the cylindrical wire. Compute the volume density from the formula

$$(48)\quad d = m/v$$

EXERCISE 153. — **To determine (a) the surface density, (b) the volume density of a sheet of metal bounded only by straight lines.**

Apparatus. — Metre stick, balance and weights, micrometer gauge.

Directions. — (*a*) Divide the given surface into triangles by drawing diagonals. Measure the base and altitude of each triangle. The sum of the areas of the

triangles is the area of the surface. On the other side of the surface draw different diagonals and make an independent determination of the surface. Average the two values. The surface density is easily found:

$$(50)\quad \textit{Surface density} = \frac{\textit{mass}}{\textit{surface}}$$

(*b*) Measure the thickness and compute the volume, which is the thickness times the surface. Then the volume density is:

$$(48)\quad d = m/v$$

EXERCISE 154. — **To make a set of weights.**

Apparatus. — Balance, weights, sheet lead, aluminum wire, shears, cutting pliers.

Directions. — For the larger weights, one gram and larger, use heavy sheet lead. First find the surface density of a rectangular piece of the lead and calculate the size needed for a 5 gram weight. Cut it too large and then pare it down gradually till the exact weight is obtained. Be very careful at the last not to take off too much. If you should get one too light use it for making 2 gram weights. A strip 4 times as long as the 5 gram weight may be used for a 20 gram and folded in 4 parts. The weights may be marked with steel dies or scratched with an awl.

For fractions of a gram use two sizes of aluminum wire, one for tenths, one for hundredths. Bend the 2 cg. piece at right angles near its middle, and the 5 to a pentagon to distinguish them.

For a set of rider weights smaller wire should be chosen.

EXERCISE 155. — To find the density of a regular solid as a sphere * or cylinder.

Apparatus. — Micrometer gauge or vernier gauge, balance and weights.

Directions. — Weigh the body as accurately as possible. Measure the dimensions of the body carefully and compute its volume. The volume of a sphere is $3.1416\ D^3/6 = .5236\ D^3$, where D is the diameter of the sphere. The volume of a cylinder is $3.1416\ r^2l$, where r is the radius and l is the length of the cylinder.

Having found the mass, m, in grams and the volume, v, in cubic centimetres, compute the density by the formula:

$$(48)\quad d = m/v$$

EXERCISE 156. — To find the density of a liquid with the graduate.

Apparatus. — Balance and weights, beaker, graduate.

The cylindrical graduate is a cylindrical glass jar of nearly uniform diameter, graduated to cubic centimetres (see Fig. 225).

Directions. — Weigh in a beaker a convenient quantity of the liquid, pour the liquid into the graduate, and weigh the beaker with what remains in it. This is a better plan than to weigh the beaker first, since some of the liquid will adhere to the beaker.

Read on the graduate the volume of the liquid, remembering that while the liquid rises on the sides of the jar, the main body of the liquid is only as high as the central part. The jar should be placed on a level

* The balls used in bicycle bearings are almost perfectly spherical.

table and the eye should be on a level with the top of the liquid to avoid parallax. As before,

$$(48) \quad d = m/v$$

EXERCISE 157. — **To find the volume and density of an irregular solid, using the graduate.**

Apparatus. — Balance and weights, graduate.

Directions. — The solid must be of a size to go into the graduate easily. Weigh the solid. Fill the graduate half full of water and note the volume. Drop the solid into the water and again note the volume. The increase in volume is of course the volume of the solid (see Fig. 225). If the solid is lighter than water it may be held below the surface by means of a needle in

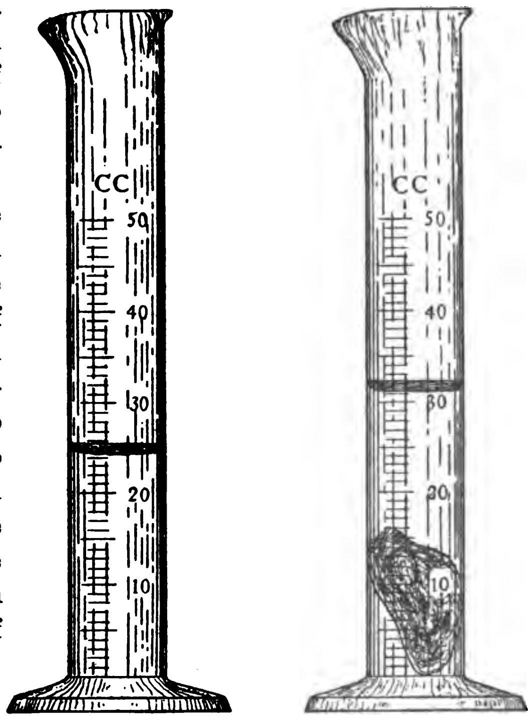


FIG. 225.

the end of a stick. The volume of the needle point may be neglected. If the solid is a crystal soluble in water it may be immersed in a saturated solution of the same substance, or it may be immersed in a liquid in which it is not soluble.

EXERCISE 158. — **To find the volume and density of an irregular solid by Archimedes' principle.**

Apparatus. — Balance and weights, tumbler, bridge, fine thread.

Directions. — By Archimedes' principle a solid when immersed in a liquid has its apparent weight diminished by an amount equal to the weight of the liquid displaced. If the liquid be water the number expressing its weight in grams expresses also its volume in cubic centimetres. But the volume of water displaced is evidently equal to the volume of the solid which displaces it, whence the loss of weight of a solid when immersed in water is the volume of the solid.

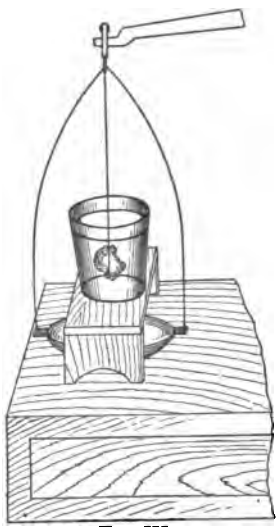


FIG. 226.

Place the bridge over the left-hand pan of the balance, taking care that it does not touch the pan (see Fig. 226). Set the tumbler upon the bridge and suspend the solid by a thread from the hook supporting the balance pan. The

body should hang free in the middle of the glass. Weigh the body and then pour water, which has been previously boiled to free it from air, into the glass till the body is completely covered and weigh again. The difference in weight will not give exactly the volume of water displaced, since a gram of water at any higher temperature than 4 degrees has expanded to a volume greater than 1 cu. cm. Note the temperature, therefore, and look up in Table 4 the volume of one gram of water at the given temperature. Multiply the loss of weight in water by the factor found in the table. This will give you v. As usual,.

$$(48)\quad d = m/v$$

Exercise 159. — **To find the density of a liquid by Archimedes' principle.**

Apparatus. — As in Exercise 158.

Directions. — Suspend a glass stopper as directed in Exercise 157. Weigh the stopper in air and in water and so find its volume. Now remove the water and fill the glass with the given liquid. A saturated solution of salt or of copper sulphate may be used. Find the loss of weight of the stopper in the given liquid. This is equal to the weight of a volume of the liquid equal to the volume of the stopper. Again,

$$(48)\quad d = m/v$$

Exercise 160. — **To find the density of a liquid with the bottle.**

Apparatus. — Balance and weights, glass-stoppered bottle.

The specific gravity bottle **(Fig. 227) furnished by dealers is usually a small Florence flask with a mark on its neck or, much better, a light bottle with a perforated ground-glass stopper, to facilitate filling the bottle without leaving air near the top. Any light glass-stoppered bottle holding from 20 to 50 c.c. will answer the purpose nearly as well.**

Directions. — Wash the bottle and dry it by putting a little alcohol in it. After shaking well so that the water will be taken up by the alcohol, pour the alcohol into the flask kept for impure alcohol. To remove what remains in the bottle rinse it with a little ether. The ether which remains will soon evaporate, leaving the bottle dry. Weigh the bottle, fill it with boiled water, note the temperature and weigh. The weight of the bottle and water less the weight of the bottle is equal to the volume, v, contained by the bottle.

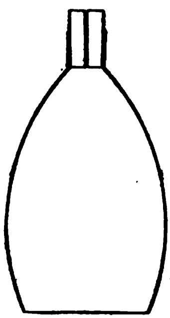


FIG. 227.

Empty and dry the bottle, fill it with the given liquid, and weigh as before. The weight is m and

$$(48) \quad d = m/v$$

EXERCISE 161. — **To find the density of a solid with the bottle.**

Apparatus. — Balance and weights, bottle.

Directions. — Find the mass, m_1, of the bottle when filled with water. Weigh the solid m, and put it into the bottle, allowing the water to overflow, wipe the bottle

and weigh again, m_2. The volume of the solid is then equal to the mass of water that overflowed, or:

$$v = m_1 + m - m_2$$

and

$$(48) \quad d = m/v$$

EXERCISE 162. — **To find the density of a liquid which does not mix with water, using the U tube.**

Apparatus. — Glass tube one metre long, bent to the shape shown in Fig. 228.

Directions. — Support the tube in a vertical position and pour enough water into it to fill it about halfway to the top. Pour into one arm enough of the given liquid to fill one arm nearly to the top, taking care that there is enough water to fill the bend of the tube. Measure the height of the liquid in each arm above the level of the surface of contact of the two liquids. It is evident that the pressure of the liquid in both tubes at the level A is equal. If the tube is of uniform diameter the masses of the columns AB and AC are equal, and if the cross section of the tube were 1 sq. cm. the volume of the given liquid would be AB cu. cm. and its weight would be AC grams. Since the ratio of the heights of the two columns is the same whatever the size of the tube may be, we may call $AB = v$ and $AC = m$, and we have

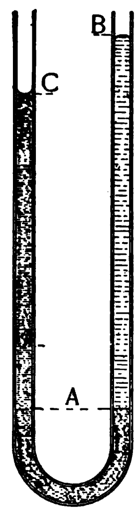


FIG. 228.

$$(48) \quad d = m/v$$

Exercise 163. — To measure the density of a liquid by the principle of the barometer.

Apparatus. — Two tumblers, two glass tubes united at the top by a **T** joint, to the stem of which is attached a piece of rubber tubing (Fig. 229).

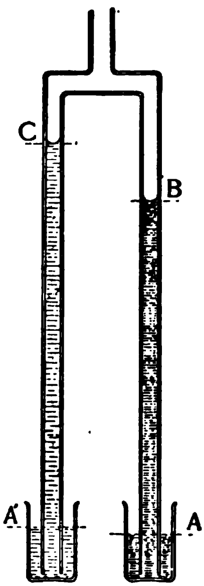


Fig. 229.

Directions. — Fill one tumbler with the liquid, the other with water. Let one of the tubes dip in each tumbler. If all of the air were now removed from the tubes, the water would rise, if possible, to a height of about 10 metres, due to the pressure of the atmosphere. Exhaust enough of the air, by applying the mouth to the rubber tube, so that the water will rise nearly to the top of the tube and close the rubber tube with a pinch cock. If the other liquid is heavier than water it will rise to a less height. Measure carefully the height of the liquid column AB above the surface of the liquid in the tumbler and of $A'C$ above the surface of the water. Then, as in Exercise 162, $A'C = m$, $AB = v$, and

$$(48) \quad d = m/v$$

Exercise 164. — To find the density of a substance lighter than water by Archimedes' principle.

Directions. — Let the student devise a method.

Exercise 165. — To find the length of a coiled wire.

Apparatus.—Same as for Exercise 158, also micrometer gauge.

Directions. — Find the volume of the wire and its diameter and compute its length.

4. Densities.

Solids.

Aluminum	2.58	Pine	0.5
Brass	8.4	Paraffine	.89
Copper	8.92	Platinum	21.5
Cork	0.24	Porcelain	2.2
Glass (common)	2.6	Quartz	2.65
,, (flint)	3.5	Sand	2.8
Gold	19.3	Silver	10.53
Ice	0.91	Steel	7.87
Iron (wrought)	7.84	Tin	7.29
Lead	11.33	Wax	.96
Oak	0.8	Zinc	7.15

Liquids (20°).

Alcohol	.789	Glycerine	1.26
Carbon bisulphide	1.29	Mercury	13.6
Ether	0.74	Sulphuric acid	1.85

Water.

t	*D*	*t*	*D*	*t*	*D*
0°	0.99988	10°	0.99974	20°	0.99827
1	0.99993	11	0.99965	21	0.99806
2	0.99997	12	0.99955	22	0.99785
3	0.99999	13	0.99943	23	0.99762
4	1.00000	14	0.99930	24	0.99738
5	0.99999	15	0.99915	25	0.99714
6	0.99997	16	0.99900	26	0.99689
7	0.99994	17	0.99884	27	0.99662
8	0.99988	18	0.99866	28	0.99635
9	0.99982	19	0.99847	29	0.99607

CHAPTER XI.

TIME.

174. Measurement of Time.—In measuring time, as in measuring length, we must be able to divide the object to be measured into equal parts. The natural divisions of time, the year, the lunar month, the day, have been subdivided into smaller units for convenience in measuring small intervals of time. Our smallest unit of time, the second, is $\frac{1}{86400}$ of a mean solar day. The period of a complete revolution of the earth upon its axis is absolutely the same from day to day and from year to year. If we are to make accurate time measurements we must employ an instrument that returns to a given starting point after equal intervals of time, or, as we say, vibrates isochronously. This condition is met if a body is held in a position of equilibrium by some force and, if, when it is displaced from that position, the force causing it to return is proportional to the displacement. This is the sort of motion which produces musical sounds, and it is known as simple harmonic motion. The vibration of a tuning fork when struck, a guitar string when picked and released, a vertical spiral spring with a weight at its lower end when pulled down and released, a vertical wire with a weight

attached when twisted and released, a common swing pendulum when the bob is drawn to one side and released, the spiral hairspring of a watch when distorted — all these are examples of simple harmonic motion.

EXERCISE 166. — **To find the time of oscillation of a simple pendulum.**

Apparatus. — Clock or watch, metre stick, small thread, ball.

Directions. — By time of *oscillation* we mean one half the time required for a complete *vibration.* The time of vibration of the *seconds* pendulum is two seconds. The pendulum should be so supported that the length of the string is the same in all parts of its swing. Fig. 230 shows a good way and a bad way of supporting the string. Adjust the pendulum to a convenient length, sit with your eye in line with the string and a mark behind it, such as the edge of the support. Set the pendulum swinging through a small arc and practise counting the swings as it passes the mark till you can look off for an instant at the watch without losing the count. Note the hour, minute, and second of the passage of the pendulum, count fifty swings, and again take the time. The difference between these two times divided by fifty will be the period of oscillation. Repeat till your observations do not vary more than one or two seconds in fifty beats. Now measure the length of the pendulum to the middle of

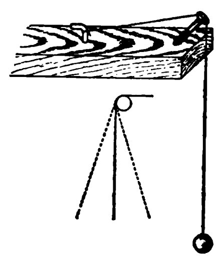


FIG. 230.

the ball with the metre stick, taking the average of the lengths to the top and bottom of the ball and compute the time of oscillation by the law of the pendulum

$$(31) \quad t = 3.1416 \sqrt{\frac{l}{g}}$$

where l is the length of the pendulum and $g = 980$.

Make the same measurements with the pendulum a little longer and again with it a few centimetres shorter.

EXERCISE 167. — **To find the time of oscillation of a spiral spring.**

Apparatus. — Jolly's balance, watch.

Directions. — Put a small weight in the pan and mark the position of the index on the scale by fastening a little paper pointer to the scale with a bit of wax. Draw the spring downward two or three centimetres and release it. Count the number of times, n, that the index passes the mark, both up and down, in one minute. The time of oscillation is then $t = 60 / n$. Count for five minutes and the time is $t = 300 / n'$, a result which is more probably correct than the former.

EXERCISE 168. — **To find the time of oscillation of a torsional pendulum.**

Apparatus. — Torsional pendulum, watch or clock, telescope.

Directions. — Fasten a bit of paper with a vertical mark upon it on the side of the weight facing you and set the telescope so that the cross hair in the eyepiece will coincide with the mark when the pendulum is at

rest (see Fig. 231). If the clock beats seconds audibly watch until the mark crosses the hair exactly on a second, then count the seconds till it passes again on an even second, while another student counts the number of beats of the pendulum. Time should be given for at least ten beats to occur. Exchange places with your assistant and repeat. Make at least five determinations apiece. Add weights to the pendulum and again determine the time.

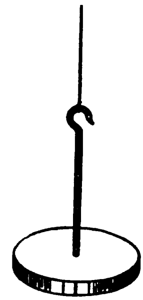


FIG. 231.

EXERCISE 169. — **To determine the acceleration due to gravity with the simple pendulum.**

Apparatus. — Metre stick with vernier, simple pendulum.

Directions. — Measure l, the length of the pendulum, in centimetres, and t, its time of oscillation, in seconds, with all possible care as directed in Exercise 166. If the pendulum is long, the error in measuring l and t will be less than with a short one. The thread must be light and the bob should not be much more than 2 cm. in diameter. A metal ball is better than a wooden one. Why? The oscillations of a simple pendulum are not strictly isochronous except for a small arc. With a pendulum one metre long the ball may swing 5 or 6 cm. to each side of the centre without introducing a perceptible error. Having determined the value of t and l, substitute them in the formula

$$(33) \quad g = (3.1416)^2\, l / t^2$$

EXERCISE 170. — **To count your pulse.**

Directions. — Lay your watch on the table before you. Place the forefinger of the right hand on the left wrist, as shown in Fig. 232. When the location of the artery is found press slightly with the forefinger and, with your eye on the watch, count the beats that occur in one minute. Repeat three times. Ask other students for the results obtained by them.

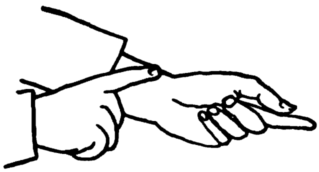


FIG. 232.

EXERCISE 171. — **To count the respiration.**

Directions. — Breathe quietly for a minute while you count the number of respirations. Try four times. Watch a student who is writing or otherwise quietly employed and count his respiration. Do the results obtained agree as closely as the time found for the pulse beat?

175. **Estimation of Time.** — It is frequently of use to be able to estimate intervals of a few seconds without the watch, as in making a photographic exposure.

EXERCISE 172. — **To estimate a short interval of time.**

Apparatus. — Watch.

Directions. — Sit, watch in hand, and count with the watch for three minutes or till you can count at about the right rate, then note the time and looking off from the watch count for fifteen seconds and again look at the watch. Record the result and repeat five times. Is

your error always on the same side? It is sometimes helpful to utter a phrase of about the right length to fill the interval between counts. The words, a thousand and one, a thousand and two, etc., may be made to serve as a rough measure of time.

EXERCISE 173. — **To find your rate of walking.**

Apparatus. — Watch.

Directions. — Count the steps you take in walking some distance you cover daily. Repeat ten times and estimate the distance from your length of step. Note the time required to walk the distance at your usual rate and also at the fastest rate which you are able to keep up. Record any difference in the direction of the wind or the like that might affect your rate.

CHAPTER XII.

FORCE.

PRESSURE OF FLUIDS.

EXERCISE 174. — **To measure the pressure of the atmosphere with the barometer.**

Apparatus. — The best form of instrument is Fortin's barometer, shown in Fig. 233. The height of the barometer is the difference in level of the two mercury surfaces *s* and *s'*. The zero of the scale is the tip of the ivory point, which projects into the cistern. Hence, that the scale may measure the height of the column, the surface *s* must be adjusted, by means of the adjusting screw at the bottom, until the ivory point touches the surface of the mercury.

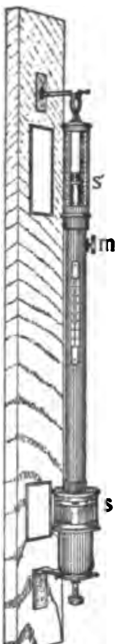


FIG. 233.

The reading is taken with the vernier, which is moved by means of the milled head, *m*, till the light reflected from the white background barely disappears at the middle point of the surface, *s'*.

To prevent parallax the vernier is graduated upon a movable tube. The eye is in the proper position to take a reading when the front lower edge of this tube appears to coincide with the back lower edge. The vernier sometimes differs from the one already used only in being divided to twentieths instead of tenths.

Directions. — Adjust the mercury in the cistern till the ivory point touches the surface. One adjustment is sufficient for the time of the exercise. Take three readings

every ten minutes for an hour. Record the time (day, hour, and minute) of each observation. Tabulate the results and plot a curve showing graphically the variations of the barometer during the hour.* Suppose your observations were as follows:

READING OF BAROMETER, OCTOBER 8.

TIME	OBSERVATIONS			AVERAGE
3:00	742.25	742.30	742.25	742.27
3:10	742.30	742.30	742.30	742.30
3:20	742.35	742.35	742.30	742.33
3:30	742.40	742.40	742.40	742.40
3:40	742.40	742.45	742.45	742.43
3:50	742.50	742.50	742.55	742.52
4:00	742.40	742.52	742.60	742.55

To plot these results in the form of a curve we let equal horizontal spaces represent equal periods of time and equal vertical spaces represent equal heights of the barometer. We divide the distance across a page of cross-section paper or any paper ruled in squares into six equal parts, as shown in Fig. 234. Then if the base line represent a height of 742.25 mm. and the top line a height of 742.55 mm., the various heights given in our table of results will fall at the corresponding heights on the paper. If we divide each vertical space into ten parts the reading for 3:00 will fall at 2.7 divisions

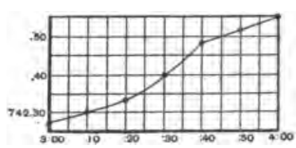


FIG. 234.

* If the barometer is not provided with a vernier follow the directions given in Exercise 132, using the metre stick and square.

above the bottom line, etc. Make a small circle at the point where the vertical line which represents by its distance to the right the time, meets the horizontal line which represents by its height the height of the barometer. Connect all these circles by a smooth curve and we have a graphic or pictorial representation of the variations of the barometer between 3 P.M. and 4 P.M. of October 8.

EXERCISE 175. — **To determine the altitude of your laboratory above the level of the sea.**

Apparatus. — Barometer, daily weather maps.

The weather maps may be seen at the post office, or on an application of the instructor through the postmaster they will be sent regularly to the laboratory, where if daily posted they may be made a source of much valuable information in the application of the principles of physics to the solution of the problems of our atmosphere. A good aneroid barometer will answer for this experiment if it has been compared with a mercurial barometer and is known to be correct.

Directions. — Read the barometer daily as nearly as possible at the hour for which the map is drawn. Locate your town as well as you can on the map and note the position of the two isobars that pass nearest to it. This will give you the reading of the barometer reduced to sea level, since all the observations of pressure made at the different stations are reduced to the pressure that would be found down a mine whose bottom was on a level with the ocean. The difference in pressure is about 1 inch for 930 feet. The difference between your reading of the barometer and the reading

obtained from the map, if both are expressed in inches, will, when multiplied by 930, give the elevation above sea level of your laboratory.

EXERCISE 176. — **To measure the pressure of the gas in the laboratory.**

Apparatus. — Open manometer and metre stick with vernier. The open manometer or pressure gauge may be made of a glass tube, bent as shown in Fig. 235. The bend of the tube is filled with water and the horizontal arm is attached by a rubber tube to the gas cock.

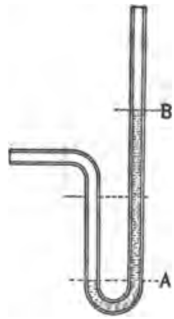


FIG. 235.

Directions. — Turn on the gas and measure the difference in level of the two surfaces *A*, *B*. If this distance be l centimetres the volume of water supported by the gas for a surface of 1 sq. cm. is l cubic centimetres and its weight is l grams. The pressure is therefore l grams or $980 \times l$ dynes per sq. cm.

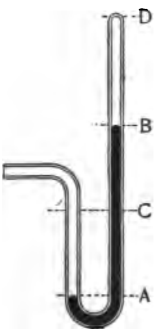


FIG. 236.

EXERCISE 177. — **To measure the pressure of the water at the hydrant and to compute the head required to produce it.**

Apparatus. — Closed manometer, metre stick, and vernier. The closed manometer consists of a bent tube with one arm closed. It has a mercury index in the bend of the tube (see Fig. 236). The open end of the tube must be fastened firmly by means of wire to the rubber tube. Both the glass tube and the rubber tube should be very strong to withstand the pressure upon them.

Directions. — Bring the mercury to the same level in both arms of the manometer by tilting it till a little mercury flows from one side to the other. Measure the length, CD, of the enclosed air column and read the barometer pressure, p.

Now turn on the water gradually till you have the full pressure and measure the length, BD.

Call $CD = v$, $BD = v'$ and $AB = l$.

Then from Boyle's law: $vp = v'p'$, whence the pressure, p', of the enclosed air is

$$(48) \quad p' = vp / v'$$

But p' is not the only pressure acting against the pressure of the water. The weight of a mercury column l cm. long is also supported by the water. The pressure of the water is therefore

$$P = p' + l \text{ cm. of mercury}$$
$$P = (p' + l) \times 13.55 \text{ grams per sq. cm.}$$
$$(49) \quad P = (p' + l) \times 13.55 \times 980 \text{ dynes per sq. cm.}$$

CAPILLARITY.

176. The Capillary Constant or coefficient of surface tension of a liquid is defined as the weight of liquid raised above its natural level per unit of length of the line bounding the surface of the liquid raised. In a capillary tube of radius r the length of the line bounding the surface is $2 \times 3.1416\ r$. If the liquid whose density is d rises to a height h the volume of the liquid raised is $3.1416\ hr^2$ and its weight is $3.1416\ hr^2d$.

The capillary constant is, then,

$$a = 3.1416\ hr^2d\ /\ 2 \times 3.1416\ r = \tfrac{1}{2}\ hrd$$

or in dynes

$$(50) \quad a = \tfrac{1}{2}\ hrd \times 980$$

EXERCISE 178. — **To find (a) the radius of a capillary tube, (b) the capillary constant of a liquid.**

Apparatus. — Clean mercury, balance, watch glass, crystallizing dish, pipette, metre stick, block (Fig. 237). This block consists of a little board with a scale, made of a piece of metre stick, attached to it. Two brass nails are driven through the board and project about 2 cm. below the top of the board.

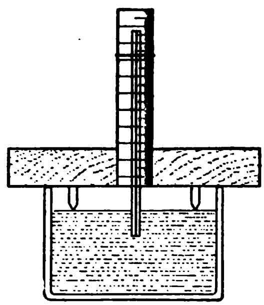


FIG. 237.

Directions. — (*a*) To measure the radius of the capillary tube.

See that the tube is perfectly dry, then draw into it a thread of mercury about 3 or 4 cm. long. This may be done by attaching a small rubber tube to one end of the glass tube and exhausting the air. Be careful not to draw the mercury into your mouth. Pinch the rubber tube to keep the mercury in place while you lay the glass tube upon a metre stick and measure the length, l, of the mercury column. Pour the mercury out into a watch glass and weigh it. The density of mercury at 20 degrees is 13.55. The volume of the mercury thread is $3.1416\ lr^2$ and its weight is $m = 13.55 \times 3.1416\ lr^2$, whence

$$(51)\quad r^2 = m \,/\, 13.55 \times 3.1416\, l$$

from which, by extracting the square root, we obtain r.

(*b*) To measure h, first clean the tube, by pouring nitric acid through it and rinsing with water. Set the block (Fig. 237) upon the crystallizing dish and fill the dish with water till the surface of the water just touches the points of the nails. Make the final adjustment with the pipette. Set the tube in front of the scale and read the height of the liquid on the scale. Raise and lower the tube. If the tube is clean the height will not be altered. Remove the tube, set the block upon the table and measure the distance from the points to the bottom of the scale. Measure at both ends and take the average. This distance added to the reading of the scale is equal to h. Substitute your values of r and h and the value for d, obtained from Table 4, in the formula, $a = \frac{1}{2}\, hrd \times 980$.

Note the temperature of the liquid and compare your result with that given in Table 5.

5. CAPILLARY CONSTANT.

TEMPERATURE	WATER	ALCOHOL
10°	77.5	24.97
15°	76.6	24.53
20°	75.7	24.09
25°	74.8	23.60

ELASTICITY.

177. Young's Modulus of Elasticity is defined as the force which would double the length of a rod

or wire of unit cross section of the substance under consideration. It may also be defined as the ratio of the stress to the strain. If a wire of length L and cross section a is elongated an amount l by a weight W the stress per unit area is $\frac{W}{a}$, while the strain per unit length is $\frac{l}{L}$. The modulus, E, is the stress divided by the strain, or $\frac{W}{a} \div \frac{l}{L}$, or

$$(52) \quad E = \frac{WL}{al}$$

The measurements are to be taken in kilograms and millimetres.

Another method of finding the modulus is by hanging weights from the middle of a rectangular bar which is supported near its ends upon knife edges, as in Fig. 239. If the vertical thickness of the bar be h, its breadth, b, and its length, L, between the triangular supports, and if a weight, W, lowers the middle of the bar a distance l it may be shown by higher mathematics that the modulus, E, is

$$(53) \quad E = \frac{1}{4} \cdot \frac{WL^3}{lbh^3}$$

The measurements are to be taken in kilograms and millimetres.

EXERCISE 179. — **To find the modulus of elasticity of a wire by stretching.**

Apparatus. — As shown in Fig. 238. The wire is supported by looping it over a strong hook in the ceiling. The ends are

twisted together at the bottom so as to support the pan for weights. To measure the elongation a lever is used consisting of a metre stick having a nail through it, around which the wires are wound two or three times, the two wires being wound in opposite directions. The nail is placed so that its centre is just on the 1 cm. division. Through the centre of the 6 cm. division a larger nail passes and rests in two supports which are attached to a block clamped to the table. A darning needle projects 6 cm. from the long end of the lever, making the long arm just twenty times as long as the short one.

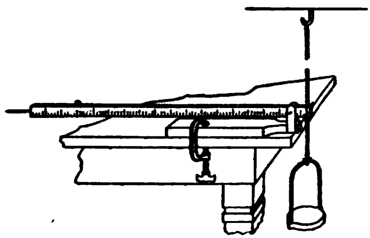


FIG. 238.

Directions. — Measure the height of the pointer above the table when enough weight is in the pan to make the wire straight. Add one kilogram and measure the height again. Add a second and third weight and measure the height of the pointer each time. Remove the weights and see if the zero reading remains unchanged. Subtract each reading from the next larger and see how they agree. If the first is much larger than the rest it should be discarded, as the wire was not yet straight. Repeat the operation twice and average all your differences. Divide the average by the ratio of the arms of the lever, and you have l, the elongation in millimetres for a force of one kilogram. Measure the length of the wire from the hook to the point of attachment. Measure the diameter of the wire

and multiply the square of the diameter by .7854 to get the area of one wire, which must, in this case, be multiplied by 2 to get a. Compute the modulus by the formula

$$(52)\quad E=\frac{WL}{al}$$

EXERCISE 180. — **To find the modulus of elasticity of a bar.**

Apparatus. — A long rectangular bar of uniform dimensions, resting on two parallel knife edges; spherometer and support, S (see Fig. 239).

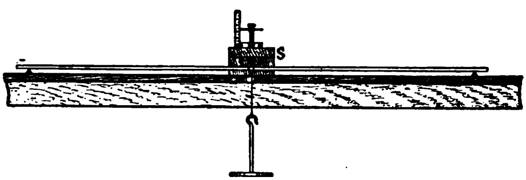


FIG. 239.

Directions. — Place the spherometer in position, with its legs passing through the small holes in support, S, which is screwed fast to the table. Take a zero reading before adding any weights. Add a kilogram weight and again take a reading with the spherometer. Add successively three weights and take a reading each time. Remove the weights one at a time and take another set of readings. The second set should agree fairly well with the first if the limit of elasticity has not been exceeded. Call the total weight added W, and the amount the bar is depressed l. Measure the vertical thickness, h, of the bar, and its breadth, b. Measure the length of the bar, L, between the knife edges.

Substitute the values obtained in the formula

$$(53) \quad E = \frac{1}{4} \frac{WL^3}{lbh^3}$$

6. Elasticity.

SUBSTANCE	MODULUS $\frac{kg.}{sq. mm.}$	BREAKING STRESS $\frac{kg.}{sq. mm.}$	
Brass	9930	60	
Copper	12449	40	
Iron	19000	83	
Wood			
Maple	1021	2.7	With the grain.
Oak	921	5.7	With the grain.
Pine	1113	4.2	With the grain.

Exercise 181. — **To measure the breaking stress of a wire.**

Apparatus. — Pail, sand, large balance, hammer, clamp, fine wire.

Directions. — Clamp a hammer to the table so that the handle will project as shown in Fig. 240. Wind one end of a fine wire several times about the hammer handle and then twist it securely fast to the clamp. Fasten the other end of the wire to the handle of the pail. The wire should be of such length that the pail will hang about an inch above the floor.

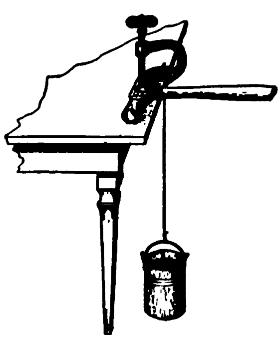


Fig. 240.

Pour sand into the pail a cupful at a time until the wire breaks. Weigh the sand and pail and take out a

single cupful and repeat, pouring the sand in slowly so as not to put in more than enough to break the wire. If it breaks where it is fastened at either end the observation must be rejected, since any bend in the wire weakens it.

Measure the diameter of the wire with the micrometer gauge and compute its cross section. The area of a circle is $3.1416\,r^2$. Calling the weight in kilograms W and the area in square millimetres of a cross section a, the breaking stress, S, is

$$(54)\quad S = \frac{W}{a}$$

FRICTION.

178. Coefficient of Friction. — The coefficient of friction is the ratio of the force required to overcome friction to the pressure on the surfaces.

EXERCISE 182. — **To find the coefficient of sliding friction.**

Apparatus. — Planed board and planed rectangular block, spring balance.

Directions. — (*a*) Lay the board upon a level table, fasten a spring balance to the middle of one end of the block by means of a string and screw eye, and lay the block on the board.

Measure the force required to keep the block sliding along the board. Weigh the block. Put a weight on the block and repeat. Care must be taken to keep the string level.

(*b*) Support one end of the board at such a height (to be found by trial) that the block, when once

started, will continue to slide down the inclined plane. If the height of the plane is AC and its base CB, the force required to overcome friction, F, is $\frac{AC}{CB}$ times the weight producing pressure, W, of the block (see Fig. 241).

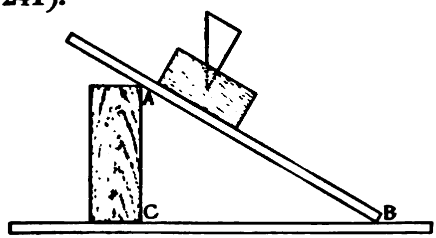


FIG. 241.

It follows that $\frac{F}{W} = \frac{AC}{CB}$ and the coefficient of friction, f, is

$$(55) \quad f = \frac{F}{W}$$

EXERCISE 183. — **To determine the efficiency of a pulley.**

Apparatus. — Large pulley and weights, spring balance.

Directions. — Weigh the weight, W, with the balance, hang the weight to one end of the rope which passes over the pulley and attach the balance to the other end. Measure the force required to keep the weight rising after it starts. Call it W'. Then the work done against friction is $W' - W = F$ and the coefficient of friction is

$$(55) \quad f = \frac{F}{W}$$

The efficiency of the pulley is

$$(56) \quad E = \frac{W}{W'}$$

if the pulley is single, since the distance through which both forces act is the same.

CHAPTER XIII.

HEAT.

TEMPERATURE.

EXERCISE 184. — **To test the zero point of a thermometer.**

Apparatus. — Thermometer, funnel, and jar.

Directions. — The zero point of a Centigrade thermometer is the temperature of melting ice. Set the funnel over the jar and fill it with bits of broken ice. When the ice has been long enough in the room so that water is dripping from the funnel, bury the thermometer in the ice as far as the zero, and leave it till it has gone as low as it will (say three minutes) and note the reading. If it should read —.3° the error is —.3° and the correction is .3°; that is, .3° must be added to the reading of this thermometer at or near 0 to give the true temperature.

EXERCISE 185. — **To test the 100° point of a thermometer.**

Apparatus. — Thermometer, flask, Bunsen burner, jacket, sheet of asbestos, support. The jacket is a large glass tube passing through a thin cork, and fitting loosely in the flask.

Directions. — The 100° point of a Centigrade thermometer is the boiling temperature of pure water. Water which contains impurities will have a boiling point slightly higher, but as the steam will be pure water it is customary to take the temperature of the steam. Fill a flask half full of water, hang the ther-

mometer, protected from the air by the jacket, from a support so that it will hang about 1 cm. above the liquid (see Fig. 242). Apply heat till the water boils. After it has boiled two or three minutes note the reading of the thermometer* and read the barometer. The thermometer should not project far above the top of the jacket. The cork should fit loosely upon the neck of the flask, so as not to confine the steam.

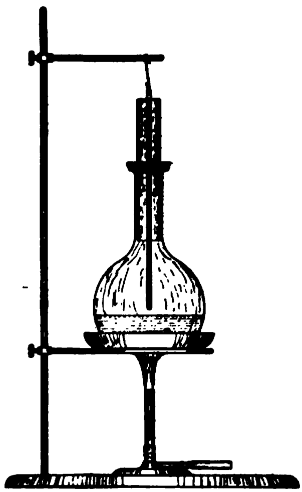


FIG. 242.

The boiling point is lower than 100° when the pressure of the air is less than 76 cm. Hence a correction must be made for the reading of the barometer. The boiling point is lowered 0.0375° C. for each millimetre decrease in the pressure of the air. The true boiling point for a pressure of b millimetres is therefore

$$(57) \quad t' = 100° - 0.0375\,(760 - b)$$

Compute t' and compare it with your observed value. The difference is the error of your thermometer near 100°.

*Should the thermometer be several degrees too low examine it carefully to see if some of the mercury is not in the little enlargement at the top. If so, it may be brought back to place by taking the thermometer in the hand and swinging it downward at arm's length.

Exercise 186. — To find the temperature of your blood.

Apparatus. — Thermometer.

Directions. — Let your assistant hold the thermometer in his mouth, keeping the lips closed, for about two or three minutes. Read the thermometer to tenths of a degree. Let your assistant take your temperature in the same way. Compare the results obtained. Compare the amount of difference in the results with the difference obtained in the pulse rate and in the respiration found in Exercises 170 and 171.

Exercise 187. — To find the melting point of a solid like paraffine.

Apparatus. — Thermometer, glass tube, beaker, Bunsen burner.

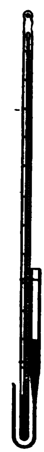


Fig. 243.

Directions. — Draw the glass tube out at one end to a capillary tube with thin walls, melt some of the solid in a tin cup and draw a portion of it into the tube. Close the tip of the tube in the flame. Make two little rubber bands by cutting bits from the end of a rubber tube, and by means of the bands attach the tube to the thermometer, as shown in Fig. 243. Heat some water in a beaker while you stir it with the thermometer, and watch carefully for the instant that the solid begins to melt, as shown by its becoming transparent. Read the thermometer, remove the flame, and watch for the liquid to solidify. The thermometer may heat and cool a little more slowly than the substance, hence the average of the two readings should be taken as the melting point

of the solid. Repeat four times, always observing the change in the smallest part of the tube.

EXPANSION.

179. Coefficient of Expansion. — The ratio of the increase in length per unit length of a substance for one degree increase of temperature is called the coefficient of linear expansion.

EXERCISE 188. — **To find the coefficient of linear expansion of a metal.**

Apparatus. — Box, spherometer, cans of thin metal about 30 cm. high and 8 cm. in diameter, thermometer, piece of plate glass. The box has holes in one end to fit the legs of the spherometer, and a large hole for the screw (see Fig. 244). Two blocks at right angles define the position of the can in the box. The can should be wrapped in several layers of flannel cloth to prevent loss of heat by radiation. It may be sewed fast and left on the can, but must not be allowed to get wet.

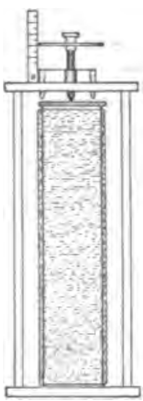


FIG. 244.

Directions. — Set the spherometer in place, measure the length, L, of the can, and fill the can to within about 5 mm. of the top with cold water. Take the temperature of the water, set the can in place, seeing that the seam is exactly over a mark on the bottom of the box and that the can is back against the blocks. Put the plate glass on top of the can, bring the spherometer screw down upon it, and read the spherometer. Raise the screw, remove the glass and can, take the temperature again, empty the can and fill

it with boiling water from a teakettle or pail, take its temperature and replace it in its former position. Again read the spherometer, remove the can, and take the temperature. The average of the temperature just before putting the can in place and just after removing it may be taken as the temperature of the metal at the time the spherometer reading was made. The difference in the two spherometer readings is the expansion, l, of the can and the difference between the temperatures of the hot and cold water is the change, t, in temperature of the metal. The coefficient of linear expansion, e, is:

$$(58) \quad e = \frac{l}{Lt}$$

We assume in this experiment that the temperature of the metal is the same as that of the water. If the metal is thin and well protected by flannel this will be very nearly the case.

Care must be taken not to spill the water in the box, as the wood will expand when wet and spoil the results.

7. COEFFICIENTS OF EXPANSION.

SUBSTANCE	COEFFICIENT	SUBSTANCE	COEFFICIENT
Aluminum	0.000023	Glass	0.000009
Brass	0.000018	Iron	0.000012
Copper	0.000017	Platinum	0.000009

SPECIFIC HEAT.

180. Heat Capacity. — Bodies differ in their *capacity for heat*, that is, in the amount of heat required to raise their temperature one degree.

The capacity of one gram of a substance divided by the capacity of one gram of water (one calorie) is called the specific heat of the substance. Water has the highest specific heat of any known substance. Hence, if we take the specific heat of water as unity the specific heats of other substances are expressed by fractions differing considerably in value for different substances.

The principle of the *method of mixtures* is as follows: If two substances at different temperatures be mixed together the cool body will take heat from the warmer till they are of the same temperature. The quantity of heat lost by the hot body must equal that given the cold body. Account must also be taken of the heat radiated to the surrounding air and of that conducted to the vessel in which the two substances are mixed and to the thermometer.

EXERCISE 189. — **To determine the specific heat of a metal by the method of mixtures.**

Apparatus. — Calorimeter, thermometer, shot or bits of wire, test tube, beaker.

Directions. — Weigh out M grams of the substances whose specific heat is sought. Weigh in the calorimeter, which may be a copper beaker or a tin cup, a mass, m, of water. If the temperature of the water is a little less than that of the room, about as much heat will be gained from the air of the room as is lost during the operation.

Put the shot or bits of metal in a test tube and set the tube in a beaker of boiling water till the metal has

had time to take the temperature of the water. Read the barometer and compute the temperature. Have ready the cup of cool water with a thermometer in it. Note the temperature of the water. Pour the metal from the test tube into the water, being careful not to let any water from the outside of the test tube fall into the cup. Now stir the shot and water with the thermometer and note the highest temperature which the mixture reaches before it begins to cool by radiation. Call the temperature of the shot T, that of the water t, and that of the mixture t'. Call the specific heat of water s. It is by definition equal to 1.

In any transfer of heat the amount of heat transferred is equal to the mass times the specific heat times the change in temperature of the substance. In this case the water has received $ms\,(t' - t)$ calories and the calorimeter and thermometer have received an amount which we call $e\,(t' - t)$, where e is the amount of water which is equivalent in heat capacity to the calorimeter and thermometer. The metal has lost $MS\,(T - t')$ calories. But the two quantities of heat are equal, hence:

$$MS\,(T - t') = (ms + e)\,(t' - t)$$

$$(59) \quad S = (ms + e)\,(t' - t) \,/\, M\,(T - t')$$

To find e, let the calorimeter and thermometer take the temperature, t_1, of the room. Pour into the calorimeter m_1 grams of water at a temperature T_1 (say 30°), stir with the thermometer, and note the temperature attained, t_1'. Then

$$m_1(T_1 - t_1') = e(t_1' - t_1)$$
$$(60) \quad e = m_1(T_1 - t_1') / (t_1' - t_1)$$

Substitute your values in the formula (59) and compare the result with the value given in Table 8.

8. SPECIFIC HEATS.

SUBSTANCE	SPECIFIC HEAT	SUBSTANCE	SPECIFIC HEAT
Aluminum	0.212	Iron	0.113
Brass	0.0891	Lead	0.032
Copper	0.0931	Mercury	0.033
Glass	0.02	Paraffine	0.589

HYGROMETRY.

181. **Hygrometry** has to do with the amount of water present at any time in the atmosphere. By the *absolute humidity* is meant the number of grams of water present in one cubic metre of air. By the *relative humidity* is meant the ratio of the amount of vapor in the air to the amount required to saturate the air at the given temperature. The *dew point* is the temperature at which the vapor now in the air would saturate the air and be deposited as dew or frost, according to whether the dew point falls above or below zero.

EXERCISE 190. — **To find the humidity and dew point with wet and dry thermometers.**

Apparatus. — Wet and dry bulb thermometers. Two thermometers of about the same size are hung a few centimetres apart. To one of them is attached a wick of cotton cloth which dips in a dish of water.

The evaporation of the water cools the bulb of the thermometer, making it read lower than the dry thermometer. The

more water there is in the air, the less evaporation there will be. When the air is saturated (relative humidity $= 1$) the two thermometers will read alike.

Directions. — See that the wick is moist. Read both thermometers, record, wait three minutes, read again, and take a third reading three minutes after the second. Currents of air may affect the readings, but the difference should be nearly constant. Call the reading of the dry thermometer t, of the wet one t'. (a) Look up in Table 9 the absolute humidity, h', for the temperature t'. It has been found that the absolute humidity, H, of the air, which is at temperature t, is, in a closed room,

$$(61) \quad H = h' - 0.96\,(t - t')$$

(b) If h be the humidity of saturated air for temperature t, as given in Table 9, then the relative humidity is

$$(62) \quad H_1 = H / h$$

(c) The dew point, T, for any absolute humidity, H, may be found from Table 9.

Compare your results (c) and (a) with the results obtained at the same hour by some other student who has used the method described in the following exercise.

EXERCISE 191. — **To find the dew point and absolute humidity of the air by cooling the air.**

Apparatus. — Beaker, water, ice water, thermometer.

Directions. — Fill the beaker one third full of water at the temperature of the room. Add ice water, a little

at a time, stirring with the thermometer. Watch carefully for the first deposit of dew on the outside of the beaker, read the thermometer, and watch for the dew to disappear and again read the thermometer. The average reading, *T*, is the dew point. The absolute humidity, *H*, corresponding to the dew point, *T*, may be found in Table 9.

9. Humidity.

t	*h*	*t*	*h*	*t*	*h*	*t*	*h*
10°	9.4	15°	12.8	20°	17.2	25°	22.9
11	10.0	16	13.6	21	18.2	26	24.2
12	10.6	17	14.5	22	19.3	27	25.6
13	11.3	18	15.1	23	20.4	28	27.0
14	12.0	19	16.2	24	21.4	29	28.6

CHAPTER XIV.

ELECTRICITY.

EXERCISE 192. — **To measure the resistance of a conductor by substitution.**

Apparatus. — Galvanometer, resistance box, switch, gravity battery.

The resistance box (Fig. 245) is a wooden case containing a set of coils of German-silver wire, R, R', etc., of known resistance, which are connected to blocks of brass, B.

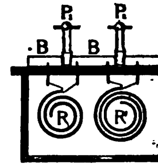


FIG. 245.

The blocks may be connected by means of the brass plugs P_1, P_2, etc. The blocks and plugs are so large that their resistance may be neglected in comparison with the resistance of the coils. When all the plugs are inserted the resistance of the box is reckoned as 0. When P_1 is removed the resistance is R. The resistance of the several coils is marked upon the top of the box.

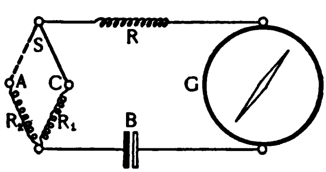


FIG. 246.

Directions. — Connect the battery, B, galvanometer, G, resistance box, R_1, unknown resistance, R_2, switch, S, and, if needed, additional resistance, R, as shown in Fig. 246. When the arm of the switch, S, is on C the current flows through R_1. When it is on A it flows through R_2. The galvanometer should be placed

with its coils north and south, the needle also north and south and reading zero. Sometimes readings are taken from a pointer at right angles to the needle. Send the current through R_2 and read the deflection on the galvanometer. If it is more than 60°, additional resistance, R, should be put in the circuit. Read both ends of the needle and take the average. Having determined the deflection produced when R_2 is in circuit, throw R_1 in circuit and vary it by changing the resistance in the box till the deflection is exactly the same as before. It is then obvious that $R_2 = R_1$.

EXERCISE 193. — **To measure the resistance of a conductor with the Wheatstone's bridge.**

Apparatus. — Galvanometer, resistance box, Wheatstone's bridge, gravity battery.

The principle of the bridge is illustrated in the typical diagram, Fig. 247. A current from the battery divides at A, part going to B through R_1, R_2, another part through R_3, R_4. It can be proved that if $R_1 : R_2 = R_3 : R_4$ no current will flow through the galvanometer between C and D. The converse is also true that if, when a current is flowing from A to B, no current flows through the galvanometer the four resistances are in the ratio named. If, then, we know R_1 and the ratio of R_4 to R_3 we can at once compute R_2. It is

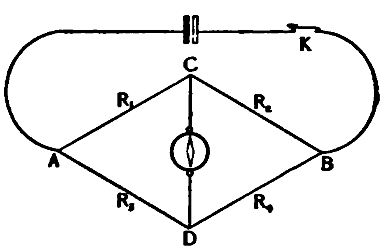


FIG. 247.

$$(63)\quad R_2 = R_1 \times R_4 / R_3$$

The bridge shown in Fig. 248 is essential in principle with the ideal bridge, Fig. 247. For convenience in adjusting the ratio of $R_4 : R_3$ these resistances are composed of a platinum or German-silver wire of uniform diameter, one metre in length, stretched between the heavy copper washers, *A* and *B*. A metre stick fastened beside the wire enables us to read at once the ratio of the parts into which the wire is divided by the contact *D*.

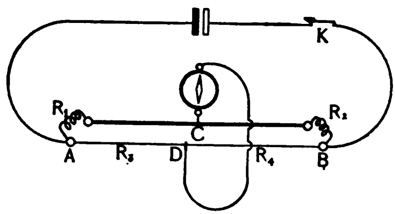


FIG. 248.

Directions.—Connect the battery to points *A*, *B*, and the galvanometer to points *C*, *D*. The coils of the galvanometer should stand north and south and the needle should read zero. The key, *K*, may be left closed when a gravity cell is in circuit. With any other battery the circuit should never be kept closed except when the readings are being taken. Connect the resistance box as R_1 and the unknown resistance as R_2. See that the contacts are clean and firm. Old wires should be scraped with a knife. If the wires are not smaller than No. 16 their resistance may be neglected. Put some resistance in the box, close *K*, and make contact with *D* at the middle of the wire. Notice which way the galvanometer deflects and move the contact first one way and then the other till you find a position such that the galvanometer deflects the other way. If you have moved far to the left of 50

the resistance R_1 is too small; if to the right, it is too large. Compute roughly what R_2 is and put about that amount in R_1. The contact will then fall near the middle. Near 50 an error in setting the contact of 1 cm. will be an error of about 2 in 50, or 4 per cent, while a like error at 90 will be an error of 2 in 10, or 20 per cent. Find the position for D where the galvanometer seems to come to zero, move it a little to the right till the needle deflects, then move it a like amount to the left and see if it deflects about the same amount in the opposite direction. When satisfied that you have the position for zero deflection read the position of D on the metre stick. If it reads from left to right the reading may be considered R_3, while 100 minus the reading is R_4, since we are concerned only with the *ratio* of R_4 to R_3, not with their absolute values.

If R_1 was less than R_2 take another observation, making R_1 greater than R_2. If possible make an observation with R_1 nearly equal to R_2.

182. Specific Resistance.—Two wires of the same length and diameter will have the same resistance if they are of the same substance, different resistances if of different substances. The resistance of unit length and unit cross section of a wire of any substance is the specific resistance of that substance. The specific resistance, r, of a wire having length L, cross section a, and resistance R is

$$(64)\quad r = \frac{Ra}{L}$$

EXERCISE 194. — **To find the specific resistance of a metal.**

Apparatus. — Wheatstone's bridge, resistance box, galvanometer, fine wire, micrometer, metre stick.

Directions. — Measure the resistance, R, length, L, and diameter of the wire. From its diameter compute its cross section, a. Compute the specific resistance from the formula

$$(64)\quad r = \frac{Ra}{L}$$

183. Temperature Coefficient of Resistance. — The resistance of a metal increases with its temperature. In the case of some alloys, like German silver, the change is slight, making them suitable for standards of resistance. In the case of carbon the resistance decreases as the temperature rises.

The temperature coefficient of resistance is the rate of increase in resistance per ohm of resistance for one degree rise in temperature. If a conductor has a resistance R at temperature t, and R' at temperature t', its coefficient between those two temperatures is

$$(65)\quad c = \frac{R' - R}{R\,(t' - t)}$$

EXERCISE 195. — **To find the temperature coefficient of resistance of a metal.**

Apparatus. — Coil of fine wire, beaker, hot water, bridge, galvanometer and resistance coils, thermometer.

Directions. — Support the coil, with the thermometer

in it, in the beaker. Measure the resistance, R, of the coil and note the temperature, t. Pour enough hot water into the beaker to cover the coil. Note the temperature, t', and measure the resistance, R'. The point of greatest difficulty will be to determine the exact temperature at the instant the reading is taken. If the coil be lifted out and the water stirred and its temperature taken at the instant the bridge is adjusted, the temperature should not be far from right.

10. Resistances.

SUBSTANCE	SPECIFIC RESISTANCE	TEMPERATURE COEFFICIENT
Aluminum	0.0000029	0.38
Copper	0.0000016	0.39
German silver	0.0000209	0.04
Iron	0.0000097	0.53

Exercise 196. — **To measure the resistance of a battery with the Wheatstone's bridge.**

Apparatus. — Bridge, small induction coil, telephone.

Directions. — If we should connect a battery as R_2 in the bridge, the current from the battery itself would flow through the galvanometer even if the bridge were balanced. To avoid this difficulty substitute for the

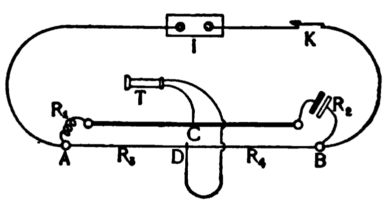


Fig. 249.

battery in the bridge a small induction coil, I (Fig.

249), giving an alternating current and for the galvanometer a telephone, T. The telephone will be affected by the alternating current from the induction coil, but not by the direct current from the battery at R_2. The induction coil should be a small one of low resistance. It may be run by a storage battery or, for a little while, by two Leclanché batteries.

Adjust the contact for silence in the telephone. In all other respects the directions given for Exercise 193 apply for this.

EXERCISE 197. — **To measure the potential difference of a battery.**

Apparatus. — Battery, ammeter, resistance box. The resistance of the ammeter, R', and battery, R'', must have been determined previously.

Directions. — Connect the battery in series with the ammeter and resistance box. With a resistance, R, in the box read from the ammeter the current, i, in ampères. By Ohm's law

$$(15a) \quad E = (R + R' + R'')\, i$$

CHAPTER XV.

SOUND.

184. Velocity of Sound. — Sound emitted by a body making n vibrations per second travels nl metres per second in a medium where the length of a wave is l metres.

EXERCISE 198. — **To find the velocity of sound in air.**

Apparatus. — Tuning fork making a known number of vibrations, long glass tube fitted with an outlet tube and pinch cock, thermometer.

Directions. — Fill the tube nearly full of water, strike the fork against the end of a heavy block of wood and hold it over the mouth of the tube. Let out a little of the water and again try the fork. When the tube sounds with a considerable volume of tone, measure the length for each trial. When the volume of sound begins to diminish try to judge which trial gave the loudest sound, fill the tube to a little above the point where the sound seemed loudest, and repeat till you are sure you have a maximum. Mark the spot, N (Fig. 250), on the tube where the sound was loudest with a rubber band or a paper label.

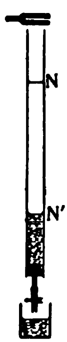


FIG. 250.

At a point, N', a little more than three times as far from the top as the first point, should be found a second point where resonance occurs.

$$(34) \quad NN' = \tfrac{1}{2} l$$
$$(33) \quad v = nl$$

from which formula we may easily compute the velocity of sound for the given temperature, since we know the number of vibrations, n, of the fork.

Note the temperature, t, of the room. Compare your result with the value given in Table 11 for that temperature.

11. VELOCITY OF SOUND

in centimetres per second.

SUBSTANCE	VELOCITY	SUBSTANCE	VELOCITY
Air 0°	33,220	Hydrogen, 0°	126,600
16°	34,179	Water, 4°	140,000
18°	34,267	Brass	350,000
20°	34,415	Aluminum	510,400
22°	34,532	Copper	356,000
24°	34,649	Glass	506,000
28°	34,881	Iron	509,300

EXERCISE 199. — **To determine the velocity of sound in a solid.**

Apparatus. — Large glass tube, long rod of wood or metal, fine cork filings, cloth and rosin, metre stick. The glass tube (see Fig. 251) is fitted at one end with a piston for varying the length of the enclosed air column. Cork filings are distributed evenly along the length of the tube. The rod is firmly clamped exactly at its middle point. To one end of the rod is cemented a disk of cardboard or cork of a size to fit loosely in the glass tube. If a hole is made in the disk to fit the rod it will be held more firmly to the rod.

Directions. — Support the tube so that the end of the rod carrying the disk projects a few centimetres within

it. With the cloth, on which has been placed some powdered rosin, rub the rod lengthwise so that a sharp note will be produced by the longitudinal vibrations. The disk will set the air in the tube in vibration, the waves will be reflected from the piston, forming stationary waves with nodes half a wave length apart. The cork filings will be disturbed at the points of greatest activity and remain at rest at the nodes. The difference will be made more evident if the tube is turned about 30°, so that the filings lie to one side of the bottom of the tube ready to slip down at the slightest disturbance. After the note has been sounded they will present the

FIG. 251.

appearance shown in Fig. 251. Adjust the piston till the proper length has been found to produce an exact number of waves and consequently a sharp indication with the cork dust. The antinodes are, like the nodes, half a wave length apart. The rod has a node at the centre and an antinode at each end; it is therefore half a wave length long. Measure l_a, the distance between two antinodes, by measuring the distance between two sharply defined antinodes near the ends of the tube and dividing the distance between by the number of spaces included. Measure l_s, the length of the rod, and note the temperature, t, of the room.

Look up v_a, the velocity of sound, for temperature, t, in Table 11.

Then since the velocity in any medium is proportional to the wave length in that medium, the velocity of sound in the rod, v_s, is

$$(38) \quad v_s = \frac{l_s}{l_a} v_a$$

EXERCISE 200. — **To find the vibration number of a tuning fork.**

Apparatus. — Two tuning forks, the vibration number of one of which is known, resonance jar, bit of soft wax.

Directions. — Pour water into the jar till it is in resonance with the standard fork. The fork to be tested must have nearly the same frequency as the standard. The jar will then be in resonance for both forks. Strike both forks at once and hold them over the jar. If they are not in perfect unison beats will be heard. Count the beats in a second by the clock.

If the standard fork has a frequency, n', and there are m beats per second, the frequency, n, of the other fork is

$$(66) \quad n = n' \pm m$$

To find whether the plus or the minus sign is to be used in this formula, reduce the time of vibration of the fork you are testing by fastening a bit of wax to one prong. If the effect is to diminish the number of beats the fork is lower than the standard, if to increase it, higher. Check this result by removing the wax from that fork and putting it on the standard fork,

CHAPTER XVI.

LIGHT.

PHOTOMETRY.

185. The Intensity of Illumination at any point varies directly with the intensity of the source of light and inversely as the square of the distance of the point from the source of light. If two surfaces, s_1, s_2, at distance d_1, d_2, from two sources of light, S_1, S_2, are equally illuminated, s_1 receiving light only from S_1 and s_2 only from S_2, then

$$S_1 : S_2 : : d_2^2 : d_1^2$$

If S_1 be a standard candle burning 7.78 grams per hour the candle power of S_2 is

$$(67) \quad S_2 = d_1^2 / d_2^2$$

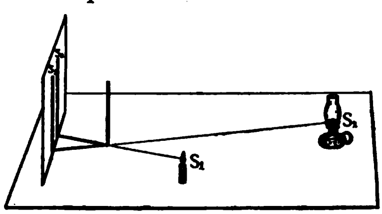


FIG. 252.

EXERCISE 201.—**To measure the candle power of a lamp with Rumford's photometer.**

Apparatus.—Paper screen, wooden rod, paraffine candle of size sixes.

Directions.—Place the rod a few centimetres in front of the screen (see Fig. 252) and set the candle about a metre from the screen. Move the lamp till the two

shadows are side by side and of equal density. Shadow s_1 (if the room be dark) is illuminated only by S_1, shadow s_2 only by S_2. Measure d_1, the distance from S_1 to s_1, and d_2, the distance from S_2 to s_2, and substitute in the formula $S_2 = d_1^2 / d_2^2$. Change d_1 and find a corresponding value for d_2. Find thus four values for S_2 and take their average.

EXERCISE 202. — **To measure the candle power of a lamp with Joly's photometer.**

Apparatus. — Candle, metre stick, paraffine blocks. Two rectangular blocks of paraffine, s_1, s_2 (Fig. 253), 2 cm. high and 1 cm. square stand side by side on a support, and are separated by a card which reaches to the front edge of the paraffine blocks.

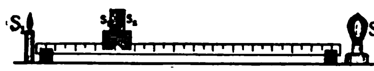


FIG. 253.

Directions. — Set the lamp and candle at opposite ends of the table and move the support till the paraffine blocks s_1, s_2, are equally illuminated. Measure d_1, d_2 and substitute in the formula

$$(67) \quad S_2 = d_1^2 / d_2^2$$

Change the distance of the lamp from the candle and make another determination.

REFRACTION.

186. Index of Refraction. — The index of refraction of a substance referred to air is the ratio of the velocity of light in air to its velocity in the given substance.

$$(35) \quad n = \frac{v_1}{v_2}$$

Exercise 203. — To find the index of refraction of glass.

Apparatus. — Piece of heavy plate glass with true edges, sheet of paper, board, three pins, compass.

Directions. — Draw on the paper two lines at right angles to each other through S (Fig. 254). Place one edge of the plate glass against one of the lines. Stick a pin vertically on the other line against the glass as at S. With the eye near the paper at E the pin may be seen on the line through the glass. Place a second pin at R and move the eye toward E' till the image of S appears in line with R and the eye and set a third pin at Q exactly in line with R and S'. Remove the glass and draw SR, the direction of the light in glass, and RQ, its direction in air. Produce QR to meet SO at S', the image of S. With a radius SO and S as a centre draw the arc OO', which is the front of a wave from S, lying wholly in glass. With S' as a centre and a radius $S'R$ draw RR', which is the front of a wave lying wholly in air.

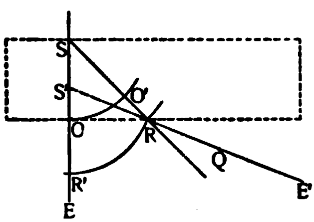


Fig. 254.

It is evident that while the wave was travelling from O' to R in glass it travelled from O to R' in air. It follows that

$$(68) \quad n = \frac{v_1}{v_2} = \frac{OR'}{O'R}$$

and we have only to measure OR' and $O'R$ with the dividers and scale.

EXERCISE 204. — **To find the index of refraction of water.**

Apparatus. — Tin cup, metre stick, try-square, clamp, shallow box, large sheet of paper, compasses, needle and block.

Directions. — Place the tin cup in one corner of the box (see Fig. 255), and clamp the metre stick in a vertical position at the other end. Stick the needle in the block, set the block over the cup with the needle projecting downward. Fill the cup with water till the surface just touches the needle. Remove the block and place the eye near the metre stick in such a position that the corner, at the bottom of the cup, at S, is just visible over the top of the cup. Place the try-square against the metre stick, and when it is in line with the top of the cup and S', the image of S, read the scale. Measure the needle to find the distance of the water from the top of the cup. Measure also the depth of the cup on the inside.

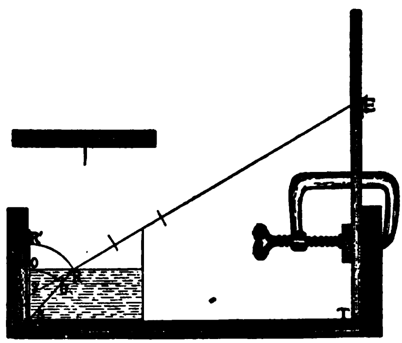


FIG. 255.

Lay off all of these dimensions on the sheet of paper as accurately as possible and draw the lines shown in Fig. 255. It is evident that

$$(68) \quad n = \frac{v_1}{v_2} = \frac{OR'}{O'R}$$

When the apparatus is in position the index of refraction of some other liquid, as alcohol, may be found by making but one measurement, *TE*, constructing the diagram and measuring *OR'* and *O'R*.

12. INDICES OF REFRACTION.

SUBSTANCE	INDEX	SUBSTANCE	INDEX
Alcohol	1.36	Glass	1.66
Carbon bisulphide	1.65	Water	1.34

LENSES.

EXERCISE 205. — **To find the focal distance of a convex lens.**

Apparatus. — Metre stick, paper or ground glass screen, candle, mirror.

Directions. — (*a*) First method. Hold the lens before an open window and move the screen toward or from it till a clear image of a distant spire, tree, or cloud is obtained on the screen (see Fig. 256). The distance *f* from the screen to the optical centre of the lens is the focal distance of the lens. The optical centre of a double-convex or double-concave lens is at the centre of the lens if both sides are of equal curvature. In a plano-convex or plano-concave lens the optical centre is at the centre of the curved surface.

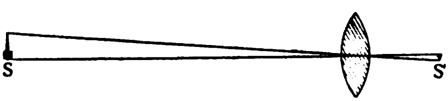


FIG. 256.

(*b*) Second method. Place a candle or other bright

object, S (Fig. 257), near a screen, s, and at one edge of it. Hold a plane mirror behind the lens and move lens and mirror together till a clear image of the candle appears on the screen. The distance of the image or object from the centre of the lens is the focal distance of the lens, for in passing once through the lens the rays are made parallel, and in passing again through the lens these parallel rays are brought together at the principal focus. If the screen were not at the principal focus the distances SO and $S'O$ could not be equal.

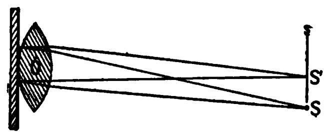


FIG. 257.

(*c*) Third method. Place the lens upon the table between the candle and the screen and move candle and screen toward or from it at the same rate till a sharp image of the candle is seen upon the screen (see Fig. 258). The candle and screen are now at the secondary foci and the distance SS' is four times the focal distance of the lens. This method does not require that we should know the optical centre of a lens if instead of making SO and $S'O$ equal we make the length of object and image equal. It will thus serve for measuring the focal distance of a system of lenses, like compound photographic lenses whose optical centre is not easily found.

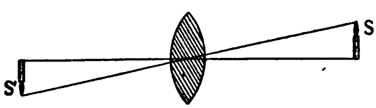


FIG. 258.

MAGNIFICATION.

EXERCISE 206. — **To measure the magnifying power (a) of a telescope, (b) of a microscope.**

Apparatus. — Cardboard scale, ruled to decimetres and centimetres, fastened to the wall about two metres from the floor so as to be above the heads of passers-by, millimetre scale, telescope, microscope.

Directions. — (*a*) Place the telescope at about the level of the eye, and at some distance from the scale. Focus the telescope on the scale by moving the eyepiece in and out. Open the left eye and try to see the scale with the left eye at the same time that you see the magnified image of the scale with the right eye. The two should overlap (see Fig. 259). Count on the scale the number of divisions covered by the magnified scale. Try five times and take the average of your observations.

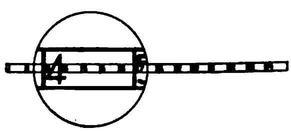


FIG. 259.

(*b*) To find the magnifying power of a microscope focus the instrument upon a millimetre scale and hold a metre stick 35 cm. from the eyepiece on or below the table of the microscope. Compare the overlapping images as in (*a*). If the instrument has a micrometer eyepiece the value of a division of the scale in the eyepiece may be determined by counting the number of divisions of the eyepiece required to cover one millimetre on the millimetre scale upon which the microscope is focused.

INDEX OF PROPER NAMES.

[The numbers refer to pages. The names are pronounced as English people pronounce them who do not know the foreign languages.]

GENERAL INDEX.*

* The references are to pages.